

APPLING GREY FORECASTING METHOD TO FORECAST THE PORTFOLIO'S RATE OF RETURN IN STOCK MARKET OF IRAN

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ABSTRACT

Stock market is one of the most important investment market, which influenced by many factors, therefore it needs a robust and accurate forecasting. In this study ,grey model used as a forecasting method and examined if it is the most reliable forecasting method in comparison of time series method. The information of portfolio's rate of return is gathered from 50 accepted companies in Tehran stock market, which were announced as the best companies last year. Mean Square of the errors (MSE) is computed by different value of α in grey model which could be varied between .1 to .9 ,to examined if $\alpha=.5$ is the best value that our model could take .Then the predictive ability of the model is compared with different type of time series based forecasting methods Experimental results confirm forecasting accuracy of grey model. Tracking signal is computed for grey model to see whether grey model forecasting is in control or not. At the last portfolio's rate of return is forecasted for next periods.

Keywords: Grey model, Stock market, Forecasting, time series

1. INTRODUCTION

In general, people are highly interested in forecasting future tendency of some events, such as investment in stock market, which is necessary to be forecasted for obtaining higher profit and reducing the investment risk. Since the prediction is mainly used to reduce the uncertainty or risk in marketing decisions ,therefore prediction accuracy is crucial.

How to select an appropriate and accurate method to predict the output forecast for enterprises is a problem of highest importance. Consequently, a method with low cost and high accuracy of the prediction has always been the goal of management decision-maker.

In recent years, researchers have developed various quantitative forecasting methods. Even though there are many forecasting methods, there does not exist a method with predictive accuracy in all circumstances.

Unfortunately, the traditional prediction model often needs to meet with large number sample or normal distribution that cannot be used to short term forecasts. In recent years, to overcome these limitations, artificial intelligence was introduced to amend traditional forecasting methods. Artificial intelligence including the artificial neural network (Hsieh,Hsiao & Yeh, 20011; Ebrahimpour, Nikoo, Masoudnia Yousefi &Ghaemi,2010),fuzzy theory(Chen,Cheng &Teoh 2008),Neural-Fuzzy system(Atsalakis &Valvanis,2009), Markov-Fourier Grey model (Hsu,Liu,Yeh &Hung ,2009) are used to solve traditional forecasting problems .

The grey theory applies to the concern of sample of small data, in which systems for the "uncertainty" , "multi-input" , "discrete data" , and" incomplete data" can effectively be addressed. It fit well with today's fast-changing industrial environment. Hence the current study proposed grey model, for assisting investors in predicting the future behavior of stock market and help them to make a rational decisions.

The grey system theory was proposed by Deng (1982). The grey model (GM) is one of the best feature in grey system theory. Generally, the grey model is written as GM(m,n) , where m is the order and n is the number of variable of the modeling equation . GM(1,1) is the most widely used and is successfully demonstrated in many application.

This study choose stock market of Tehran as a database and gathered information of portfolio's rate of return from 50 accepted firms in Tehran stock market, which were announced as the best firms last year. At first different value of α examined for our model to see if $\alpha=.5$ is the best value that α could take to produce the most accurate model, and after that the model with the least error will be chosen and, at the last the predictive ability

of the models are compared with different type of forecasting in time series, which here is Naive method, Simple Average, Moving Average, Single Exponential Smoothing.

The rest of this paper is organized as follows. section 2 describes grey model which is used as a forecasting method in stock market. Section 3 presents the results and compares of the proposed model with $\alpha=5$ and the other value of α , and compares of grey model accuracy with time series forecasting method and also results of tracking signals which are computed for grey model, and at the last of this section future return rate is forecasted . section 4 summarized the findings.

2. METHODOLOGY

The procedure of the original GM(1,1) model will be briefly illustrated in the following.

Assume that $x^{(0)}$ stands for the raw data series of portfolio's rate of return ,namely,

$$x^{(0)} = (x^{(0)}(1), x^{(0)}(2), x^{(0)}(3), \dots, x^{(0)}(n)) \quad (1)$$

Where n is the sample size. By 1-AGO(one time Accumulated Generating Operation) $x^{(0)}$,the preprocessed series, $x^{(1)}$

$$x^{(1)} = (x^{(1)}(1), x^{(1)}(2), x^{(1)}(3), \dots, x^{(1)}(n)) \quad (2)$$

Where $x^{(1)}(k) = \sum_{m=1}^k x^{(0)}(m)$, for $k=1,2,\dots,n$

By mean operation on $x^{(1)}$,the series $Z^{(1)}$

$$\text{Where } Z^{(1)} = (Z^{(1)}(1), z^{(1)}(2), \dots, z^{(1)}(n)) \quad (3)$$

Thus ,from grey system theory (Deng,1988)the grey differential equation of GM(1,1) and its whitening equation are obtained, respectively, as follows:

$$x^{(0)}(k) + aZ^{(1)}(k) = b, \quad k = 2,3,\dots,n$$

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = b$$

(4)

Where a and b is the developing coefficient and grey input, respectively. Let $\hat{\theta}$ be the parameters vector. By using least squares method (Hsia,1979), the parameters a and b can be obtain as

$$\hat{\theta} = (X^T X)^{-1} X^T Y = \begin{bmatrix} a \\ b \end{bmatrix} \quad (5)$$

$$\text{Where } X = \begin{bmatrix} -Z^{(1)}(2) & 1 \\ -Z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -Z^{(1)}(n) & 1 \end{bmatrix} \quad Y = \begin{bmatrix} X^{(0)}(2) \\ X^{(0)}(3) \\ \vdots \\ X^{(0)}(n) \end{bmatrix} \quad (6)$$

And x denotes the accumulated matrix and Y represent the constant vector. The approximate relation can be obtained by substituting the $\hat{\theta}$ into the differential equation, and solving equation (4)

$$\hat{x}^{(1)}(k+1) = \left[X^{(0)}(1) - \frac{b}{a} \right] e^{-ak} + \frac{b}{a} \quad (7)$$

Where $\hat{x}^{(0)}(1)=x^{(0)}(1)$. by IAGO (inverse AGO) equation (7) the recovered value $\hat{x}^{(0)}(k)$ is:

$$\hat{x}^{(0)}(k) = \hat{x}^{(1)}(k) - \hat{x}^{(1)}(k-1) = (1 - e^{-a}) \left[X^{(0)}(1) - \frac{b}{a} \right] e^{-a(k-1)} \quad (8)$$

Given $k=1,2,3,\dots,n$, the predictive value is

$$\hat{x}^{(0)} = (\hat{x}(1), \hat{x}^{(0)}(2), \hat{x}^{(0)}(3), \dots, \hat{x}^{(0)}(n)) \quad (9)$$

Finally, the forecasting data is further examined to see if it meets the residual error checking procedure. Usually, the following equation is utilized (Deng, 1988), where the residual error $e(k)$ is between the actual $x^{(0)}(k)$ and the predicted $\hat{x}^{(0)}(k)$.

$$e(k) = \left| \frac{x^{(o)}(k) - \hat{x}^{(o)}(k)}{x^{(o)}(k)} \right| \times 100\% \quad (10)$$

But because in this case, the value of some raw data is zero, we use the square sum of the errors for comparing the errors as below:

$$s = \varepsilon^T \varepsilon = \begin{bmatrix} \varepsilon(2) \\ \varepsilon(3) \\ \vdots \\ \varepsilon(n) \end{bmatrix}^T \cdot \begin{bmatrix} \varepsilon(2) \\ \varepsilon(3) \\ \vdots \\ \varepsilon(n) \end{bmatrix} \quad (11)$$

3. EMPIRICAL ANALYSIS

In order to show the predictive ability of GM (1,1) model, its performance and comparison with other models(time series),we choose the Tehran stock market as a sample.

We use the information of portfolio's rate of return in 50 accepted firms in Tehran stock market. This information is gathered from March 2007 to February 2010 .

Some information of some company was missed. And we could just gather information of 45 company.

3.1.Different value of back ground

At first we compute mean square of errors of grey model by different value of α , which could differ between .1 to .9(.1< α <.9). α is called generation coefficient (or weight). When $\alpha>.5$ the generation is said to have “emphasis more on new and less on old information”. When $\alpha<.5$, the generation is said to have “emphasis more on old and less on new information”. And when $\alpha=.5$, the generation we get the best result.(liu & lin,2006) Mean square of errors computed by different value of α which started from .1,the procedure stops whenever errors increase in comparison of last error value which computed with lower value of α . By this way we can be sure that, $\alpha=.5$ gives us the best results and the most accurate model. The results are shown in table 1.

| Firm | $\alpha=.1$ | $\alpha=.2$ | $\alpha=.3$ | $\alpha=.4$ | $\alpha=.5$ | $\alpha=.6$ |
|------|---------------|---------------|---------------|---------------|---------------|---------------|
| A | 9961.2 | 9954.4 | 9949.2 | 9945.7 | 9943 | 9943.6 |
| B | 3120.9 | 3119.2 | 3118.3 | 3117.6 | 3117.5 | 3118 |
| C | 4745.9 | 4744.2 | 4742.7 | 4742 | 4740 | 4742 |
| D | 2695.3 | 2694.4 | 2693.7 | 2693.2 | 2690 | 2693.2 |
| E | 3543.1 | 3541.9 | 3508 | 3540 | 3570 | |
| F | 3055 | 3054 | 3053.2 | 3052.8 | 3052 | 3052.8 |
| G | 3395.3 | 3393.9 | 3392.5 | 3391.7 | 3391.1 | 3390.9 |
| H | 5010.9 | 5008.9 | 5007.1 | 5006.3 | 5005 | 5006.3 |
| I | 3698.2 | 3702.1 | 3706.5 | | 3716 | |
| J | 25893 | 25879 | 25871 | 25867 | 25867 | 25872 |
| K | 3448.8 | 2447.7 | 3446.9 | 3446.5 | 3440 | 3446.4 |
| L | 6326.5 | 6307.6 | 6312.3 | | 6324.8 | |
| M | 3565.6 | 3564.3 | 3563.9 | 3564.2 | 3565.6 | |
| N | 544.10 | 543.00 | 542.30 | 541.78 | 541.51 | 541.52 |
| O | 4719.2 | 4716.9 | 4715.2 | 4714.2 | 4713.8 | 47140 |
| P | 4671 | 4666.9 | 4663.8 | 4661.6 | 4660 | 4660 |
| Q | 1190.2 | 1192.4 | | | 1200 | |
| R | 4335.9 | 4334.2 | 4332.7 | 4332 | 4260 | 4331.9 |
| S | 7308.8 | 7304.3 | 7300.7 | 7298.8 | 7298 | 7299.1 |
| T | 5250.5 | 5248 | 5246.3 | 5245.2 | 5279.5 | |
| U | 9262.7 | 9258.2 | 9255 | 9253.2 | 9250 | 9253.8 |
| V | 3874.7 | 3873 | 3871.8 | 3870.7 | 3870 | 3870.8 |
| W | 1060.4 | 1060.3 | 1060.2 | 1060.3 | 1059 | 1061 |
| X | 1857.3 | 1250.9 | 1249.5 | 1248.8 | 1248.6 | 1249 |
| Y | 2562.1 | 2561 | 2560.3 | 2565.8 | 2559.9 | 2559.8 |
| Z | 2690.4 | 2689.3 | 2688.6 | 2688.4 | 2688.1 | 2688.2 |
| AA | | | | | 3410 | |
| BB | 3690.3 | 3689.1 | 3688.4 | 3687.9 | 3680 | 3688.7 |
| CC | 3211.5 | 3226.2 | 3210 | 3209.6 | 3200 | 3209.6 |

| | | | | | | |
|-----------|---------------|----------|----------|---------------|----------------|----------|
| DD | 9018.8 | 9011.5 | 9006.4 | 9003.6 | 9002 | 9003.6 |
| EE | 924.25 | 923.1271 | 922.3832 | 921.8877 | 921.725 | 921.8671 |
| FF | 2067.5 | 2066.7 | 2066.1 | 2065.8 | 2065 | 2065.9 |
| GG | 1255.5 | 1255.6 | | | 1257.2 | |
| HH | 2079.6 | 2078.6 | 2077.8 | 2077.4 | 2077 | 2077.6 |
| II | 2895.7 | 2894.5 | 2893.9 | 2893.3 | 2893.2 | 2893.4 |
| JJ | 2843.1 | 2841.7 | 2840.9 | 2840.4 | 2840 | 2840.5 |
| KK | 5696 | 5665.8 | 5636.1 | 5629.6 | 5720 | |
| LL | 3000.8 | 2998.8 | 2997 | 2995.8 | 3008 | |
| MM | 1107.7 | 1107.3 | 1106.9 | 1106.7 | 1106.6 | 1106.6 |
| NN | 4454.1 | 4452.2 | 4450.6 | 4449.6 | 4449.2 | 4449.4 |
| OO | 3240 | 3239.2 | 3238.6 | 3238.3 | 3328.3 | |
| PP | 5179.9 | 5179 | 5179 | 5179 | 5079.7 | 5177.3 |
| QQ | 1613.8 | 1613.2 | 1612.7 | 1612.5 | 1612.4 | 1612.5 |
| RR | 5001.5 | 5000.3 | 4999.4 | 4998.9 | 4990 | 4998.9 |
| SS | 8473.5 | 8466.9 | 8462.2 | 8459 | 8450 | 8458 |

Table1

As it shown in bold, there are just few companies (5 firms from 45) whose errors with the other value of α are less than what we have computed with the value of $\alpha=.5$. so the results confirm that when it is difficult to measure the reliability of new and old information due to a shortage of needed background information , the method of generation of equal weight or generation of “no preference “ gives us better result.

3.2. Comparison of GM (1,1) with the other models

To examine whether the proposed model has made improvement in forecasting accuracy, the mean square of the errors defined in Eq (11), is employed as a evaluation criterion for the forecasting performance of grey model and comparison with the other models, which are time series model here.(Naive, Simple Average , Moving Average ,Single Exponential Smoothing). As we mentioned before we could not use the residual error $e(k)$ as a evaluation criterion, because the portfolio’s rate of return in some company in some months was zero. The results are shown in table 2 .

Table 2

| Firm | Naive | Simple Average | Moving Average | Single Exponential Smoothing | GM(1,1) |
|----------|----------|----------------|-----------------|------------------------------|-----------------|
| A | 379.9111 | 260.1798 | 279.68 | 259.123 | 225.9773 |
| B | 1496.16 | 796.2273 | 927.6875 | 858.7611 | 70.85227 |
| C | 177.0773 | 116.8865 | 974.0655 | 113.1108 | 107.7273 |
| D | 155.9702 | 67.10227 | 92.27514 | 82.24468 | 61.13636 |
| E | 167.0032 | 98.09498 | 119.206 | 236.467 | 80.45455 |
| F | 126.3532 | 71.25659 | 71.78109 | 76.85709 | 69.36364 |
| G | 127.0703 | 87.24909 | 100.6853 | 95.67861 | 77.06591 |
| H | 240.5305 | 128.5725 | 163.7755 | 139.5756 | 113.75 |
| I | 181.5904 | 77.125 | 43.64255 | 94.33664 | 84.05 |
| J | 1083.049 | 645.2045 | 846.4955 | 674.9161 | 587.8864 |
| K | 185.4861 | 88.06659 | 107.3971 | 94.11236 | 78.18182 |
| L | 341.4793 | 130.002 | 110.378 | 183.4363 | 143.7455 |
| M | 160.6766 | 87.88955 | 108.5902 | 92.46275 | 80.99773 |
| N | 68.18387 | 36.12533 | 42.84597 | 51.29547 | 36.10067 |
| O | 236.2933 | 125.2488 | 151.2079 | 127.697 | 107.561 |
| P | 190.5133 | 121.1048 | 126.2431 | 124.2766 | 105.9091 |
| Q | 125.0894 | 46.22818 | 39.00205 | 526.3186 | 27.05 |
| R | 157.2619 | 111.999 | 132.124 | 111.3686 | 103.9024 |
| S | 281.4505 | 192.0283 | 234.3811 | 180.3747 | 165.8636 |
| T | 196.0786 | 138.8283 | 183.5117 | 153.0255 | 128.7683 |
| U | 288.7777 | 226.3609 | 256.8027 | 230.6707 | 210.2273 |
| V | 177.6989 | 96.365 | 115.977 | 93.11395 | 87.95455 |
| W | 48.40377 | 28.25818 | 33.75352 | 25.87718 | 24.06818 |
| X | 104.1044 | 95.64481 | 59.79034 | 84.35656 | 78.0375 |
| Y | 290.3561 | 134.255 | 71.5123 | 1844.016 | 58.17955 |
| Z | 84.29095 | 569.1768 | 59.32485 | 55.54941 | 65.56341 |

| | | | | | | |
|-----------|----------|----------|----------|----------|--|-----------------|
| AA | 1441.206 | 838.9327 | 1113.668 | 857.3757 | | 775.9773 |
| BB | 120.8151 | 97.0825 | 95.80602 | 92.06709 | | 83.63636 |
| CC | 123.5421 | 80.20386 | 98.73995 | 99.12602 | | 72.72727 |
| DD | 427.2098 | 218.9625 | 293.1125 | 212.4843 | | 204.5909 |
| EE | 104.084 | 51.30318 | 50.74782 | 67.464 | | 41.89659 |
| FF | 103.4005 | 59.89045 | 65.30234 | 53.03127 | | 46.93182 |
| GG | 66.30182 | 34.34523 | 39.47159 | 32.32095 | | 28.53409 |
| HH | 78.47632 | 54.84182 | 59.11041 | 58.57345 | | 47.20455 |
| II | 143.9774 | 72.02909 | 94.03995 | 79.05884 | | 65.75455 |
| JJ | 106.0909 | 95.53235 | 116.8493 | 97.12471 | | 83.52941 |
| KK | 2593.718 | 1420.526 | 1863.201 | 1438.723 | | 127.9455 |
| LL | 101.9635 | 76.45614 | 85.44248 | 82.63164 | | 68.08636 |
| MM | 55.34564 | 28.7325 | 34.50777 | 30.70661 | | 25 |
| NN | 153.3529 | 112.7168 | 125.5467 | 117.1564 | | 101.1182 |
| OO | 127.5058 | 79.81295 | 82.96307 | 93.01141 | | 73.59773 |
| PP | 175.0922 | 137.6459 | 162.3418 | 174.0962 | | 115.4477 |
| QQ | 83.63027 | 40.64091 | 54.22693 | 47.16705 | | 36.64545 |
| RR | 213.4828 | 126.2975 | 131.98 | 124.7506 | | 113.4091 |
| SS | 387.725 | 216.9759 | 263.9334 | 228.1511 | | 192.2182 |

As it is shown in table 2, the mean square of errors in grey model is less than the other models, except in two cases(I,L) which the errors of Moving Average method are less than the others and one case(z) which the error of Single Exponential Smoothing is less than the others. So the results confirm the predictive ability of grey model forecasting.

3.3. Tracking Signal

Tracking signal indicates if the forecast is consistently biased high or low. It is computed by dividing the Running Sum of Forecast Errors(RSFE) by the cumulative mean absolute deviation ,or MAD:

$$TS = \frac{RSFE}{MAD}$$

The tracking signal is recomputed each period , with updated , running value of cumulative error and MAD. The movement of the tracking signal is compared to control limits ,as long as the tracking signal is within these limits, the forecast is in control. Control limits are between $\pm 4MAD$ or $\pm 3\sqrt{MSE}$.

It computed for grey model to see if this model forecasting is in control or not. The result are shown in tables 3,4 and 5.

As it is seen, the forecast is completely in control and the plots of tracking signals visually display this.

Table 3

| Firm | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|----------|------|------|------|------|-------|------|------|------|------|------|------|------|-------|------|
| A | -3 | 1.8 | 1.3 | 0.7 | 0.1 | -0.5 | -1.3 | -2.1 | -3.0 | -3.8 | -4.7 | - | -6.6 | -7.5 |
| | | | | | | | | | | | | | 5.69 | |
| B | -0.6 | -1.7 | 1.8 | 2.3 | 1.7 | 1.6 | 0.6 | -0.4 | -1.2 | -2.4 | -2.5 | -2.8 | -3.8 | -5.1 |
| C | -3 | -4 | 1.05 | 2.8 | 1.09 | 1.0 | 0.1 | 0.7 | - | - | -0.2 | -1.7 | -0.4 | 1.1 |
| | | | | | | | | | 0.34 | 1.19 | | | | |
| D | -2.5 | 1.8 | 2.4 | 1.6 | 1.5 | 0.41 | 1.1 | -2.2 | -3.9 | -5.0 | -5.4 | -7.3 | -8.9 | -3.0 |
| E | -8.0 | 0.79 | 1.02 | 4.4 | 1.8 | 1.2 | 0.87 | 0.28 | -0.8 | -2.3 | -3.5 | -3.1 | -2.5 | -1.3 |
| F | 3 | 3.01 | 3.2 | 3.2 | 2.7 | 2.4 | 2.9 | 2.14 | 1.4 | 0.19 | -0.7 | -0.3 | -1.3 | - |
| | | | | | | | | | | | | | 1.16 | |
| G | -3 | -4 | -5 | -6 | -7 | -8 | -9 | -10 | -11 | -8.1 | -9.2 | -6.0 | -4.0 | - |
| | | | | | | | | | | | | | 5.07 | |
| H | -3 | -0.1 | 1.2 | 1.3 | -0.29 | -1.5 | -2.4 | -3.2 | 0.65 | -0.3 | 1.96 | 1.03 | 1.13 | 3.0 |
| I | 2.08 | 2.99 | 2.70 | 3.05 | 2.99 | 2.85 | 2.25 | 1.64 | 0.99 | 0.26 | -0.4 | -1.2 | -1.7 | - |
| | | | | | | | | | | | | | 1.93 | |
| J | -3 | -4 | -5 | -6 | -7 | -8 | -9 | -10 | -11 | -12 | -13 | -0.9 | -0.74 | - |
| | | | | | | | | | | | | | 1.03 | |
| K | -3 | - | -4.2 | 0.4 | -0.2 | -0.5 | 0.07 | -1.4 | -0.6 | -2.2 | -3.1 | -1.3 | -1.34 | -1.5 |

| | 3.21 | | | | | | | | | | | | | | |
|-----------|------|------|------|------|-------|------|------|------|------|------|------|-------|-------|------|--|
| L | 1.7 | 2.8 | 2.8 | 2.5 | 2.0 | 1.6 | 1.8 | 2.8 | 2.5 | 2.08 | 1.3 | 0.78 | 0.3 | -0.5 | |
| M | 3 | 4 | 3.2 | 2.8 | 4.1 | 3.9 | 3.8 | 3.0 | 3.4 | 3.3 | 2.8 | 3.0 | 1.4 | 3.7 | |
| N | | | | | | | | | | | | | | | |
| O | | | | -3 | -4 | -5 | -6 | -5.3 | -6.3 | -7.3 | 0.6 | 0.1 | -0.4 | -1.1 | |
| P | -0.9 | 3.4 | 3.1 | 2.3 | 1.4 | 3.1 | 4.3 | 4.9 | 5.1 | 4.8 | 3.8 | 2.9 | 2.1 | 1.7 | |
| Q | -3 | -2.9 | -0.3 | 1.6 | 0.002 | 2.8 | 1.5 | 2.7 | 2.7 | 1.3 | 1.3 | -0.1 | 2.1 | 2.6 | |
| R | -3 | -4 | -5 | -6 | -5.6 | -5.5 | -5.8 | -6.8 | -8.0 | -8.5 | - | -10 | -7.0 | 0.1 | |
| | | | | | | | | | | | 9.75 | | | | |
| S | -3 | 2.1 | 3.3 | 2.8 | 2.6 | 2.0 | 2.3 | 3.3 | 2.5 | 1.6 | 0.7 | - | -0.9 | 0.7 | |
| T | -3 | -0.4 | -0.5 | 0.2 | 1.1 | 0.3 | 0.6 | 1.9 | 1.4 | - | -0.9 | -1.3 | -0.05 | 1.1 | |
| U | -3 | -2.5 | -3.7 | -4.8 | -5.9 | -6.9 | -8 | -6.1 | -7.2 | -8.3 | -9.3 | - | -11.2 | -3.5 | |
| V | -3 | -4 | -5 | -6 | -7 | -8 | -9 | -5.0 | -6.1 | -7.3 | -8.4 | -9.0 | -10 | -1.5 | |
| W | -3 | 1.5 | 2.11 | 1.58 | 2.03 | 2.52 | 3.17 | 4.76 | 6.0 | 4.96 | 3.8 | 4.5 | 3.25 | 2.22 | |
| X | | | | | | | | | | | | | | | |
| Y | 1.4 | 2.41 | 1.61 | 1.48 | 1.59 | 2.12 | 1.88 | 1.06 | 0.04 | 0.69 | - | -1.3 | -0.84 | 0.29 | |
| Z | -0.8 | -0.2 | 0.44 | -1.5 | -2.51 | - | - | - | - | - | - | - | -2.75 | - | |
| AA | -3 | -4 | -5 | -6 | -7 | -8 | -9 | -10 | -3.7 | - | - | -5.6 | -5.87 | - | |
| | | | | | | 4.15 | 5.44 | 6.67 | 7.75 | 8.91 | 10.1 | 3.82 | | 1.21 | |
| BB | 3 | 2.45 | 1.86 | 1.44 | 0.72 | - | 1.0 | 0.58 | - | - | -1.7 | - | -1.09 | -1.4 | |
| | | | | | | 0.01 | | | 0.28 | 0.79 | | 0.55 | | | |
| CC | -0.9 | -0.5 | - | 0.74 | 1.44 | 1.16 | - | - | - | - | - | - | -6.72 | - | |
| | | | | | | 0.75 | 2.22 | 3.46 | 4.23 | 4.81 | 5.44 | 6.69 | | 1.48 | |
| DD | -1.3 | -2.7 | - | - | -3.22 | - | - | - | - | - | - | - | 2.13 | 2.94 | |
| | | | | | | 4.32 | 4.42 | 5.54 | 6.84 | 7.81 | 7.86 | 9.08 | | | |
| EE | | | | | | | | | | | | | | | |
| FF | 0.71 | 0.52 | 0.19 | 0.54 | 0.36 | -0.5 | - | - | - | 1.35 | 1.63 | 1.83 | 1.52 | 1.54 | |
| | | | | | | | 1.21 | 0.38 | 0.19 | | | | | | |
| GG | 0.04 | - | - | - | -2.94 | -4.3 | 0.42 | -1.0 | -1.4 | - | -4.2 | 1.18 | -0.04 | 1.5 | |
| | | 0.07 | 0.05 | 0.86 | | | | | 2.72 | | | | | | |
| HH | -3 | -4 | -5 | -6 | -4.2 | -5.0 | -6.0 | 1.97 | 2.97 | 2.45 | 2.0 | 2.28 | 2.54 | 2.34 | |
| II | -2.3 | -3.6 | - | - | -4.24 | -5.6 | -2.4 | -2.6 | -3.4 | -4.6 | -5.9 | -6.6 | 0.25 | -1.2 | |
| | | | | | | 1.87 | 2.91 | | | | | | | | |
| JJ | | | | | | | | | | -3 | -4 | -1.68 | - | 0.26 | |
| KK | -3 | -0.7 | 4.22 | 5.27 | 6.18 | 7.19 | 7.48 | 8.33 | 8.61 | 9.42 | 10.0 | 10.5 | 11.1 | 11.9 | |
| LL | -3 | -4 | -5 | -6 | -5.11 | - | - | -7.9 | -8.0 | -3.4 | - | - | -1.73 | - | |
| | | | | | | 5.94 | 6.79 | | | 3.99 | 0.28 | | | 2.77 | |
| MM | -3 | -4 | 0.78 | - | 1.85 | 2.2 | 1.66 | 1.87 | 1.01 | 0.4 | - | - | 4.3 | 3.66 | |
| | | | | 0.52 | | | | | | 0.27 | 0.62 | | | | |
| NN | -3 | -4 | -5 | -6 | -4.53 | - | - | 2.62 | 2.4 | 1.83 | 1.6 | 1.4 | 1.81 | 0.85 | |
| | | | | | | 5.22 | 1.91 | | | | | | | | |
| OO | -0.5 | 0.54 | 1.59 | 2.8 | 1.37 | 0.89 | 0.37 | - | - | -3.7 | - | -5.45 | - | 6.13 | |
| | | | | | | | 0.22 | 1.32 | 2.35 | | 4.74 | | | | |
| PP | -0.2 | 2.23 | 3.11 | 2.45 | 2.73 | 0.86 | - | - | -2.3 | - | - | - | -2.89 | - | |
| | | | | | | 0.33 | 1.42 | | 3.58 | 5.08 | 5.98 | | 1.26 | | |
| QQ | -3 | 1.47 | - | - | -1.84 | - | - | - | - | - | - | -1.4 | 0.005 | - | |
| | | | | 0.36 | 1.93 | | 3.04 | 4.33 | 1.34 | 1.83 | 2.12 | 0.19 | | 0.63 | |
| RR | 0.16 | 2.44 | 1.99 | 1.85 | 2.98 | 1.56 | 0.26 | 1.47 | 1.11 | 0.08 | - | - | 0.37 | 0.57 | |
| | | | | | | | 0.57 | | 1.66 | | | | | | |
| SS | 0.1 | - | - | -3.5 | -4.58 | - | 1.12 | 1.34 | 1.14 | 0.37 | 3.17 | 2.56 | 2.3 | 2.08 | |
| | | | | | | 5.64 | | | | | | | | | |

Table 4

| Firm | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 |
|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| A | -8.4 | -9.4 | -4.0 | -2.8 | -3.6 | -2.9 | -3.2 | -3.0 | -2.5 | -2.3 | 0.4 | 3.0 | 2.3 | 2.8 | 2.6 |
| B | -3.4 | -3.1 | -2.7 | -3.8 | -3.5 | -3.7 | -4.5 | -3.5 | -3.6 | -4.6 | 3.8 | 4.0 | 3.4 | 3.3 | 2.9 |
| C | 0.56 | -0.5 | -2.1 | -1.9 | 1.03 | 2.1 | 0.67 | - | -1.3 | -2.7 | - | -3.1 | -0.9 | -1.6 | - |
| | | | | | | | 0.12 | | | 2.13 | | | | | 3.01 |
| D | -2.5 | -3.4 | -4.2 | 0.36 | - | -1.6 | - | -3.6 | - | 1.4 | -0.4 | 2.6 | 1.3 | 1.8 | 0.22 |
| | | | | | 0.22 | | 2.02 | | 0.16 | | | | | | |
| E | 0.9 | 1.1 | 2.2 | 1.4 | 1.2 | 0.7 | 1.9 | 0.4 | -0.2 | 0.02 | -0.5 | -1.8 | -2.6 | -4.0 | 0.4 |
| F | - | -0.8 | -2.0 | -2.4 | -3.4 | -4.3 | -4.6 | -5.0 | -2.3 | -3.3 | -2.6 | -3.5 | -3.3 | -4.3 | -2.5 |
| | | 0.02 | | | | | | | | | | | | | |
| G | - | -4.8 | -2.5 | -1.9 | -2.9 | -3.7 | -3.0 | -0.4 | -1.3 | -1.7 | -1.0 | 1.05 | 1.06 | 1.41 | 4.15 |
| | | 6.07 | | | | | | | | | | | | | |
| H | 4.06 | 3.84 | 3.05 | 1.9 | 1.6 | 0.8 | 0.1 | -0.4 | -0.7 | -1.6 | -2.2 | -1.9 | -2.7 | -2.4 | 0.7 |
| I | -2.3 | -1.4 | -2.1 | -2.6 | -1.8 | -2.2 | -2.8 | -3.4 | -4.2 | -4.4 | -4.9 | - | -6.2 | -3.2 | -3.0 |
| | | | | | | | | | | | 5.62 | | | | |
| J | - | -2.0 | 1.7 | 2.98 | 2.5 | 2.5 | 3.7 | 5.0 | 6 | 5.2 | 4.4 | 4.3 | 4.9 | 4.3 | 4.7 |
| | | 1.57 | | | | | | | | | | | | | |
| K | -1.5 | -1.5 | - | 3.31 | 3.14 | 3.47 | 2.96 | 3.13 | 3.83 | 4.32 | 3.35 | 2.06 | 2.16 | 2.81 | 1.45 |
| | | 2.41 | | | | | | | | | | | | | |
| L | -1.1 | -1.8 | -2.5 | - | - | - | - | - | - | -4.8 | - | -3.6 | - | - | - |
| | | | 2.89 | 3.65 | 4.45 | 5.25 | 4.28 | 4.08 | | 5.11 | | 1.43 | 1.46 | 0.29 | |
| M | 2.4 | 1.24 | 0.09 | - | - | - | - | - | -6.1 | - | -8 | - | 0.2 | - | -0.9 |
| | | | 1.01 | 2.08 | 3.11 | 4.13 | 5.12 | | 7.05 | | 0.08 | | 0.39 | | |
| N | | | | | | | | | | | | 3 | 0.98 | 0.19 | |
| O | -0.6 | -0.8 | - | -2.7 | -2.6 | -2.1 | 0.14 | -0.6 | -0.8 | -1.1 | -1.7 | -1.8 | 0.17 | -0.7 | -1.0 |
| | | 1.68 | | | | | | | | | | | | | |
| P | 3.7 | 2.6 | 1.4 | 1.5 | 0.4 | -0.8 | -1.5 | -2.2 | -3.2 | -3.7 | -5.2 | -6.6 | - | -9.1 | - |
| | | | | | | | | | | | | 8.05 | | | 10.4 |
| Q | 2.2 | 2.4 | 3.8 | 2.4 | 3.09 | 3.2 | 3.5 | 3.3 | 5 | 6.5 | 6.2 | 4.4 | 6.9 | 6.2 | 4.8 |
| R | 1.15 | 1.8 | 0.7 | 0.99 | - | -1.3 | 0.19 | 0.26 | 3.04 | 3.1 | 3.2 | 3.2 | 5.3 | 5.01 | 7.1 |
| | | | | | 0.17 | | | | | | | | | | |
| S | 0.52 | 1.8 | 0.9 | 0.2 | -0.5 | -1.5 | -2.4 | -3.4 | -4.3 | -5.2 | -6.2 | -3.5 | -1.5 | -2.1 | - |
| | | | | | | | | | | | | | 1.09 | | |
| T | 4.04 | 3.9 | 3.9 | 4.98 | 3.5 | 2.03 | 0.8 | 1.5 | 2.1 | 1.2 | 0.2 | -1.1 | - | 4.9 | 6.8 |
| | | | | | | | | | | | | 2.01 | | | |
| U | -3.5 | -3.3 | - | 4.03 | -4.3 | -3.3 | 1.9 | 4.7 | 5.2 | 5.9 | 5.5 | 5.1 | 6.2 | 6.3 | 6.6 |
| | | | | | | | | | | | | | | | 6.3 |
| V | -2.7 | -1.2 | 0.19 | 2.5 | 3.2 | 2.1 | 1.7 | 0.9 | - | -0.4 | -1.5 | 1.3 | 2.2 | 4.4 | 3.5 |
| | | | | | | | | 0.05 | | | | | | | |
| W | 1.81 | 0.96 | 1.36 | 0.08 | -0.9 | -1.2 | - | 2.54 | -2.7 | 0.24 | -0.5 | -1.7 | -3.3 | -2.2 | -1.2 |
| | | | | | | | | | | | | | | | -2.0 |
| X | | | | | | | | | | | | | 2.06 | 2.96 | 1.66 |
| Y | -0.3 | -1.4 | -2.2 | -0.8 | -0.5 | - | - | - | - | -1.1 | 0.6 | 1.33 | 0.71 | 0.01 | 0.7 |
| | | | | | 0.04 | 0.31 | 1.15 | 0.95 | | | | | | | |
| Z | -0.8 | 1.39 | 2.5 | 3.79 | 2.35 | 3.02 | 3.7 | 3.09 | 3.12 | 4.16 | 3.09 | 2.6 | 0.85 | - | - |
| | | | | | | | | | | | | 0.03 | 0.47 | | |
| AA | -5.5 | - | - | - | - | - | - | - | 1.61 | 2.86 | 2.34 | 1.42 | 0.43 | 2.5 | 1.77 |
| | | 6.27 | 7.43 | 8.58 | 9.73 | 10.7 | 11.5 | 12.2 | | | | | | | |
| BB | -1.7 | -2.4 | -3.2 | -2.8 | -2.4 | - | - | - | - | -1.4 | 1.43 | 2.98 | 3.32 | 2.7 | 2.17 |
| | | | | | | 2.62 | 3.48 | 4.17 | 3.23 | | | | | | |
| CC | 1.67 | 2.31 | 1.53 | 0.86 | 1.89 | 0.59 | - | -0.5 | 0.60 | - | - | - | - | - | 2.93 |
| | | | | | | 0.49 | | | | 0.68 | 1.06 | 0.67 | 0.54 | 0.04 | |
| DD | 2.67 | 2.2 | 2.35 | 1.86 | 1.62 | 1.52 | 0.63 | 0.3 | 3.27 | 2.39 | 2.16 | 1.96 | 0.92 | 0.03 | 1.84 |
| EE | | | | | | | | | | -2.1 | -3.2 | -4.3 | -0.7 | 0.29 | -1.0 |
| FF | -2.2 | -2.9 | -2.7 | 0.85 | 1.68 | 1.64 | 0.97 | 0.36 | 0.69 | 0.49 | - | -0.9 | -1.5 | -0.6 | - |
| | | | | | | | | | 0.47 | | | | | 0.16 | |
| GG | 1.05 | 0.84 | - | 0.14 | 0.63 | -0.1 | - | - | 0.23 | 2.45 | 3.32 | 2.37 | 3.29 | 2.05 | 2.27 |
| | | | | | 1.64 | 1.95 | 2.99 | | | | | | | | |
| HH | 1.77 | 1.46 | 1.52 | 0.72 | 1.75 | 1.93 | 2.36 | 2.15 | 1.55 | 0.88 | 0.92 | 0.6 | 0.35 | - | -0.7 |

| | | | | | | | | | | | | | | | | | |
|-----------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| | | | | | | | | | | | | | | | | | 0.08 |
| II | 1.11 | 0.13 | -1.0 | 1.13 | 1.43 | 0.55 | - | 0.25 | 1.25 | 1.05 | - | -1.8 | 1.74 | 4.1 | 3.34 | 2.34 | 4.14 |
| JJ | -0.8 | -2.4 | -3.9 | -4.6 | -5.7 | - | -0.6 | - | - | - | - | 2.55 | 3.43 | 4.41 | 3.57 | 2.45 | |
| KK | 13.0 | 12.5 | 13.4 | 13.6 | 14.3 | 14.0 | 13.9 | 14.6 | 12.7 | 10.8 | 11.4 | 9.35 | 9.02 | 6.83 | 7.24 | | |
| LL | -3.2 | -4.2 | 0.13 | 1.51 | -0.1 | - | - | - | - | -6.4 | -7.9 | -9.2 | -2.2 | -1.4 | 3.61 | | |
| MM | 2.3 | 2.64 | 2.18 | 0.97 | 0.69 | - | - | - | - | -3.3 | -2.3 | -3.9 | -4.9 | -2.9 | -1.4 | | |
| NN | -0.2 | -0.9 | -1.7 | -2.3 | -2.9 | - | - | - | - | -2.2 | 1.33 | 1.8 | 0.65 | 1.66 | 1.93 | | |
| OO | - | - | - | - | - | - | - | - | - | 1.81 | 2.29 | 3.68 | 3.72 | 5.01 | 6.74 | 6.06 | |
| | 5.37 | 6.35 | 6.07 | 7.04 | 2.81 | 2.29 | 3.59 | 2.84 | | | | | | | | | |
| PP | - | - | -1.5 | 0.69 | 1.5 | 0.43 | - | - | - | - | - | 0.19 | 1.27 | 2.33 | 2.27 | 1.51 | |
| | 1.12 | 0.03 | | | | | 0.64 | 1.12 | 2.22 | 3.09 | | | | | | | |
| QQ | -1.9 | 1.89 | 3.08 | 3.07 | 2.11 | 1.07 | 1.33 | 1.02 | 1.56 | 1.05 | - | 1.28 | 1.57 | 2.49 | 1.17 | | |
| | | | | | | | | | | 0.41 | | | | | | | |
| RR | - | 1.15 | - | - | - | - | - | - | -5.8 | - | - | - | - | - | - | - | |
| | 0.07 | | 0.13 | 1.38 | 2.03 | 3.32 | 3.97 | 5.22 | | 6.25 | 3.85 | 3.07 | 1.68 | 2.84 | 1.41 | | |
| SS | 1.39 | 1.32 | 1.03 | 0.34 | - | - | - | - | - | - | - | - | - | - | - | - | |
| | | | | | 0.65 | 1.55 | 2.55 | 3.25 | 3.69 | 2.34 | 2.13 | 2.94 | 3.85 | 4.15 | 0.54 | | |

Table5

| Firm | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | | |
|----------|------|-------|------|------|------|------|------|------|-------|------|-------|-------|-------|-------|--|--|
| A | 3.1 | 3.9 | 3.9 | 4.3 | 4.0 | 3.1 | 3.0 | 2.9 | 2.0 | 1.6 | 0.9 | 0.85 | 0.5 | -0.4 | | |
| B | 5.6 | 5.9 | 5.3 | 5.2 | 4.4 | 4.5 | 4.0 | 3.7 | 2.9 | 1.9 | 2.6 | 1.9 | 1.0 | 0.3 | | |
| C | -3.5 | -2.6 | -2.2 | -3.6 | -2.8 | 1.7 | 2.3 | 4.91 | 5.1 | 3.90 | 2.8 | 1.5 | 0.9 | -0.07 | | |
| D | 4.55 | 3.5 | 2.16 | 1.07 | 1.25 | 0.20 | - | - | 1.9 | 0.69 | 0.05 | 0.07 | 0.26 | -0.2 | | |
| | | | | | | | | 1.02 | 1.92 | | | | | | | |
| E | 2.11 | 0.97 | 0.81 | 0.15 | -0.7 | -1.8 | 2.47 | 1.63 | 1.83 | 2.15 | 2.1 | 1.3 | 0.3 | 0.3 | | |
| F | - | -0.71 | -0.6 | 1.4 | 1.46 | 1.51 | 0.76 | 0.82 | -0.22 | 1.88 | 1.53 | 1.19 | 0.29 | 0.04 | | |
| | 1.31 | | | | | | | | | | | | | | | |
| G | 4.3 | 3.5 | 3.09 | 2. | 1.73 | 1.51 | 2.4 | 1.48 | 0.75 | - | -1.1 | -0.7 | -0.09 | -1.5 | | |
| | | | | | | | | 0.01 | | | | | | | | |
| H | 1.2 | 0.5 | 0.27 | 0.30 | 0.61 | - | - | -1.7 | -2.2 | 1.27 | 0.29 | 1.09 | 0.07 | 0.4 | | |
| | | | | | 0.49 | 1.04 | | | | | | | | | | |
| I | -1.6 | -0.8 | -1.3 | -0.6 | 0.5 | 0.12 | 0.01 | 1.3 | 2.8 | 4.0 | 3.9 | 3.6 | 3.3 | 3.3 | | |
| J | 3.92 | 4.04 | 3.58 | 3.6 | 3.81 | 4.1 | 3.87 | 3.73 | 3.06 | 2.6 | 2.01 | 1.65 | 1 | 0.22 | | |
| K | 0.97 | 0.5 | 0.48 | 1.05 | 1.5 | 0.73 | 0.23 | 0.02 | -0.1 | 1.23 | 2.4 | 2.57 | 1.02 | 0.61 | | |
| L | - | -0.51 | - | -0.7 | -1.3 | -1.3 | - | - | 1.11 | 0.67 | 1.04 | 2.67 | 2.27 | 2.98 | | |
| | 0.29 | | | | | | 1.84 | 2.16 | | | | | | | | |
| M | -1.5 | -1.29 | -1.8 | -2.2 | -2.4 | -2.8 | - | - | -1.14 | - | -0.48 | -1.07 | -1.2 | 2.07 | | |
| | | | | | | 3.36 | 3.88 | | 1.16 | | | | | | | |
| N | 0.97 | -0.28 | - | - | - | -1.2 | 0.31 | - | -0.73 | - | 2.07 | 1.99 | 1.56 | 0.87 | | |
| | 0.36 | 1.31 | 2.02 | | | | 0.08 | | 1.89 | | | | | | | |
| O | 2.49 | 2.14 | 1.02 | 1.12 | 2.7 | 2.4 | 3.9 | 4.4 | 4.16 | 3.29 | 2.4 | 2.38 | 1.26 | -0.1 | | |
| P | - | -5.2 | -3.1 | -2.5 | -3.3 | 1.6 | 3.5 | 3.02 | 3.48 | 3.11 | 2.7 | 1.98 | 0.91 | -0.11 | | |
| | 4.53 | | | | | | | | | | | | | | | |
| Q | 6.7 | 5.4 | 5.07 | 4.9 | 6.17 | 6.7 | 7.01 | 6.9 | 7.7 | 8.5 | 10.1 | 10.5 | 8.59 | 8.98 | | |
| R | 8.27 | 7.3 | 7.6 | 6.8 | 6.7 | 5.6 | 4.2 | 4.15 | 5.15 | 4.12 | 3.45 | 3.12 | 1.91 | 1.71 | | |
| S | - | -0.08 | - | -0.6 | - | 1.13 | 0.53 | 0.54 | 0.7 | 0.92 | 0.98 | 1 | 0.78 | 0.14 | | |
| | 0.92 | 0.53 | 0.97 | | | | | | | | | | | | | |
| T | 6.02 | 5.49 | 4.57 | 3.47 | 2.68 | 1.95 | 1.29 | 0.61 | 2.05 | 2.33 | 2.28 | 1.96 | 1.08 | -0.02 | | |
| U | 5.58 | 4.9 | 4.76 | 3.9 | 3.78 | 3.68 | 2.8 | 1.8 | 2.05 | 1.46 | 1.61 | 0.56 | -0.48 | 0.12 | | |
| V | 3.5 | 3.2 | 1.9 | 1.8 | 1.4 | 0.3 | -0.8 | -1.1 | 0.25 | -0.2 | 2.4 | 1.3 | 1.8 | 0.37 | | |
| W | - | -3.2 | - | -4.6 | -2.3 | -3.8 | -3.0 | -3.6 | -0.68 | 1.24 | 1.17 | 1.09 | 0.29 | -0.58 | | |
| | 2.06 | 3.62 | | | | | | | | | | | | | | |

| | | | | | | | | | | | | | | |
|-----------|------|-------|------|------|------|------|------|------|-------|------|-------|-------|-------|-------|
| X | 0.54 | -0.86 | - | -2.9 | -3.3 | - | - | 0.02 | 2.59 | 2.89 | 2.84 | 1.93 | 1.46 | 0.841 |
| Y | - | 0.6 | - | - | - | - | - | - | 0.31 | 1.0 | 0.19 | 1.29 | 1.07 | 1.08 |
| | 0.10 | | 0.49 | 0.90 | 0.99 | 1.57 | 2.05 | 0.52 | | | | | | |
| Z | 1.74 | 1.42 | 1.46 | 4.63 | 4.16 | 2.11 | 0.75 | - | -0.57 | -1.3 | -1.26 | 2.11 | 3.74 | 1.6 |
| | | | | | | | | 1.26 | | | | | | |
| AA | 2.2 | 1.194 | 0.12 | 0.08 | - | - | - | - | -2.05 | - | -1.0 | -0.66 | -1.53 | -2.37 |
| | | | | | 0.01 | 0.07 | 0.21 | 0.99 | | 0.47 | | | | |
| BB | 2.15 | 2.89 | 2.29 | 1.79 | 1.74 | 1.64 | 1.13 | 1.32 | 1.28 | 2.6 | 1.87 | 1.71 | 1.2 | 0.91 |
| CC | 2.4 | 2.97 | 2.72 | 3.01 | 2.6 | 2.2 | 1.03 | 2.26 | 1.37 | 0.82 | 2.57 | 1.15 | 1.42 | 0.14 |
| DD | 1.2 | 0.25 | - | -1.6 | 2.89 | 2.85 | 2.28 | 1.81 | 1.35 | 1.07 | 2.06 | 1.93 | 0.89 | 0.15 |
| | | | | 0.69 | | | | | | | | | | |
| EE | 0.94 | 0.5 | - | -0.7 | - | - | 0.95 | - | 3.89 | 2.47 | 1.31 | 1.88 | 0.96 | 0.15 |
| | | | | 0.12 | 0.52 | 0.07 | | 0.23 | | | | | | |
| FF | 0.7 | 0.52 | 0.19 | 0.54 | 0.36 | -0.5 | - | - | -0.19 | 1.3 | 1.63 | 1.83 | 1.52 | 1.54 |
| | | | | | | | 1.21 | 0.38 | | | | | | |
| GG | 0.94 | 2.21 | 0.58 | 1.02 | 0.94 | - | 1.82 | 1.11 | 1.33 | 3.16 | 4.4 | 5.5 | 4.86 | 2.52 |
| | | | | | 0.51 | | | | | | | | | |
| HH | - | -0.37 | 1.23 | 3.53 | 4.75 | 5.19 | 5.51 | 4.69 | 4.09 | 3.22 | 2.57 | 2.66 | 1.73 | 0.82 |
| | | 0.81 | | | | | | | | | | | | |
| II | 3.44 | 2.88 | 2.03 | 1.8 | 1.15 | 0.71 | 0.33 | - | -0.82 | 1.1 | 1.4 | 0.99 | 0.67 | 0.13 |
| | | | | | | | 0.36 | | | | | | | |
| JJ | 2.52 | 1.75 | 1.26 | 0.53 | 0.25 | 1.7 | 1.84 | 2.54 | 1.78 | 1.71 | 1.92 | 1.33 | 0.46 | 0.32 |
| KK | 9.39 | 11.28 | 9.1 | 7.79 | 8.78 | 6.77 | 3.94 | 4.18 | 5.31 | 22.1 | 20.45 | 18.34 | 16.81 | 14.94 |
| LL | 4.73 | 3.24 | 1.76 | 0.21 | - | - | - | - | -4.26 | -5.5 | -6.22 | -5.2 | -3.02 | -3.97 |
| | | | | | 1.27 | 2.22 | 3.09 | 4.09 | | | | | | |
| MM | -1.0 | -1.7 | - | - | -2.9 | - | 0.93 | 0.79 | 1.17 | - | 2.14 | 2.07 | 0.4 | -0.27 |
| | | | | 0.48 | 1.64 | 0.62 | | 0.43 | | | | | | |
| NN | 2.21 | 1.32 | 0.32 | 2.96 | 2.99 | 1.78 | 1.2 | 0.5 | 0.001 | 0.66 | 0.48 | 0.2 | -0.31 | -0.22 |
| OO | 5.1 | 4.42 | 3.93 | 3.31 | 2.68 | 2.38 | 2.08 | 1.03 | 0.35 | 0.20 | 0.35 | 0.87 | 0.45 | 0.24 |
| PP | 1.07 | 1.22 | 1.36 | 0.36 | 0.63 | 2.43 | 2.83 | 2.08 | 2.44 | 3.33 | 2.93 | 2.58 | 1.55 | 0.41 |
| QQ | 1.15 | 0.04 | - | - | - | -1.0 | 2.27 | 1.74 | 0.71 | - | 2.61 | 1.44 | 1.29 | 0.29 |
| | | | | 0.79 | 1.34 | 2.62 | | 0.26 | | | | | | |
| RR | 0.38 | 2.54 | 3.0 | 2.96 | 2.99 | 2.71 | 2.59 | 3.97 | 3.61 | 2.66 | 1.76 | 1.79 | 0.99 | 0.2 |
| SS | 0.48 | 0.07 | 2.7 | 2.91 | 1.83 | 0.85 | 0.1 | - | -1.2 | 0.38 | -0.73 | -1.53 | -0.94 | -0.57 |
| | | | | | | | 0.69 | | | | | | | |

3.4. T-test exam

The two sample t-test simply tests whether or not two independent populations have different mean values on some measure. Here the mean square of errors in grey model as a measure of comparison are compared with mean square of errors in time series forecasting models. As it is seen in tables 7 and 8 the null hypothesis has been accepted (“p-value” is bigger than .05), which can be concluded, grey model has the same mean square of errors with moving average and simple average forecasting models. But in comparison of grey model with naive and single exponential smoothing forecasting models(table6 and9), the null hypothesis rejected, which mean they have a different mean square of errors. As it seen in table 7 and 11 for both models, lower and upper interval difference have the negative value which can be concluded that mean square of errors in grey model are less than mean square of errors in naive and single exponential smoothing model. So with more emphasis ,we can accept that grey model has the better ability in forecasting in comparison of naive and single exponential smoothing models.

Table 6

| GM(1,1) VS naive | t-test for equality of means | | | |
|----------------------------|------------------------------|---|----------|--|
| | Sig.(2-tailed) | 95% Confidence Interval of the Difference | | |
| | | Lower | Upper | |
| Equal variance assumed | .012 | -186.5145 | -42.3959 | |
| Equal variance not assumed | .013 | -186.5145 | -40.9358 | |

Table 7

| GM(1,1) VS simple average | t-test for equality of means | | |
|----------------------------|------------------------------|---|---------|
| | Sig.(2-tailed) | 95% Confidence Interval of the Difference | |
| | | Lower | Upper |
| Equal variance assumed | .118 | -157.4106 | 18.1313 |
| Equal variance not assumed | .12 | -157.8403 | 18.5610 |

Table 8

| GM(1,1) VS moving average | t-test for equality of means | | |
|----------------------------|------------------------------|---|--------|
| | Sig.(2-tailed) | 95% Confidence Interval of the Difference | |
| | | Lower | Upper |
| Equal variance assumed | .059 | -221.9907 | 4.3240 |
| Equal variance not assumed | .061 | -222.8945 | 5.2278 |

Table 9

| GM(1,1) VS Single Exponential Smoothing | t-test for equality of means | | |
|---|------------------------------|---|----------|
| | Sig.(2-tailed) | 95% Confidence Interval of the Difference | |
| | | Lower | Upper |
| Equal variance assumed | .044 | -232.7626 | -3.21150 |
| Equal variance not assumed | .046 | -233.6962 | -2.2778 |

3.5. Forecast next year's portfolio's rate of return

After the predictive ability of grey model ,in comparison of time series based forecasting methods, was proved ,we forecast portfolio's rate of return for 12 months later.(march 2010 to march 2011).The result is shown in table 10.

| Firm | 2010 | | | | | | | | | | | 2011 | | | |
|------|-------|-------|--------|--------|-------|-------|-------|-------|-------|-------|-------|-------|-----|-----|-----|
| | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec | Jan | Feb | Mar | Apr | May | Jun |
| A | 12.37 | 12.46 | 12.56 | 12.65 | 12.74 | 12.84 | 12.93 | 13.03 | 13.13 | 13.22 | 13.30 | 13.42 | | | |
| B | 16.37 | 16.23 | 16.098 | 15.960 | 15.82 | 15.68 | 15.55 | 15.42 | 15.28 | 15.15 | 15.02 | 14.89 | | | |
| C | 11.43 | 11.44 | 11.46 | 11.48 | 11.50 | 11.52 | 11.54 | 11.56 | 11.58 | 11.60 | 11.62 | 11.64 | | | |
| D | 8.78 | 8.80 | 8.82 | 8.85 | 8.87 | 8.89 | 8.91 | 8.93 | 8.96 | 8.98 | 9.00 | 9.02 | | | |
| E | 8.09 | 8.04 | 8.00 | 7.95 | 7.91 | 7.86 | 7.82 | 7.77 | 7.73 | 7.68 | 7.64 | 7.60 | | | |
| F | 9.31 | 9.30 | 9.28 | 9.27 | 9.25 | 9.24 | 9.23 | 9.21 | 9.20 | 9.19 | 9.17 | 9.16 | | | |
| G | 11.29 | 11.44 | 11.60 | 11.75 | 11.91 | 12.06 | 12.22 | 12.39 | 12.55 | 12.72 | 12.89 | 13.06 | | | |
| H | 8.45 | 8.38 | 8.31 | 8.24 | 8.17 | 8.10 | 8.03 | 7.97 | 7.90 | 7.83 | 7.77 | 7.70 | | | |
| I | 2.17 | 2.09 | 2.01 | 1.93 | 1.86 | 1.79 | 1.72 | 1.65 | 1.59 | 1.53 | 1.47 | 1.41 | | | |
| J | 12.56 | 12.44 | 12.33 | 12.21 | 12.10 | 11.98 | 11.87 | 11.76 | 11.65 | 11.54 | 11.43 | 11.33 | | | |
| K | 8.59 | 8.52 | 8.45 | 8.38 | 8.31 | 8.24 | 8.17 | 8.10 | 8.03 | 7.96 | 7.89 | 7.83 | | | |
| L | 3.42 | 3.31 | 3.19 | 3.09 | 2.98 | 2.88 | 2.78 | 2.69 | 2.60 | 2.51 | 2.42 | 2.34 | | | |
| M | 3.04 | 2.96 | 2.89 | 2.81 | 2.74 | 2.67 | 2.61 | 2.54 | 2.48 | 2.41 | 2.35 | 2.29 | | | |
| N | 5.13 | 4.99 | 4.85 | 4.72 | 4.59 | 4.47 | 4.35 | 4.23 | 4.12 | 4.01 | 3.90 | 3.79 | | | |
| O | 9.01 | 9.04 | 9.06 | 9.09 | 9.12 | 9.14 | 9.17 | 9.20 | 9.22 | 9.25 | 9.28 | 9.30 | | | |
| P | 12.54 | 12.79 | 13.04 | 13.30 | 13.56 | 13.83 | 14.10 | 14.38 | 14.67 | 14.96 | 15.25 | 15.56 | | | |
| Q | 7.63 | 7.69 | 7.75 | 7.81 | 7.87 | 7.92 | 7.98 | 8.04 | 8.10 | 8.17 | 8.23 | 8.29 | | | |
| R | 10.84 | 10.94 | 11.04 | 11.15 | 11.26 | 11.36 | 11.47 | 11.58 | 11.69 | 11.80 | 11.92 | 12.03 | | | |
| S | 9.92 | 9.89 | 9.86 | 9.83 | 9.80 | 9.77 | 9.73 | 9.70 | 9.67 | 9.64 | 9.61 | 9.58 | | | |
| T | 9.20 | 9.19 | 9.19 | 9.18 | 9.17 | 9.16 | 9.15 | 9.15 | 9.14 | 9.13 | 9.12 | 9.11 | | | |
| U | 9.61 | 9.59 | 9.58 | 9.56 | 9.54 | 9.52 | 9.51 | 9.49 | 9.47 | 9.46 | 9.44 | 9.42 | | | |
| V | 11.03 | 11.18 | 11.33 | 11.48 | 11.64 | 11.80 | 11.96 | 12.12 | 12.28 | 12.45 | 12.62 | 12.79 | | | |
| W | 8.44 | 8.54 | 8.65 | 8.76 | 8.87 | 8.98 | 9.10 | 9.21 | 9.33 | 9.45 | 9.57 | 9.69 | | | |
| X | 6.22 | 6.02 | 5.84 | 5.65 | 5.48 | 5.31 | 5.14 | 4.98 | 4.82 | 4.67 | 4.53 | 4.39 | | | |
| Y | 6.51 | 6.42 | 6.34 | 6.26 | 6.18 | 6.10 | 6.02 | 5.95 | 5.87 | 5.80 | 5.72 | 5.65 | | | |
| Z | 13.53 | 13.82 | 14.12 | 14.42 | 14.72 | 15.04 | 15.36 | 15.69 | 16.02 | 16.36 | 16.71 | 34.51 | | | |
| AA | 22.46 | 22.89 | 23.32 | 23.76 | 24.21 | 24.67 | 25.13 | 25.61 | 26.09 | 26.58 | 27.08 | 27.60 | | | |

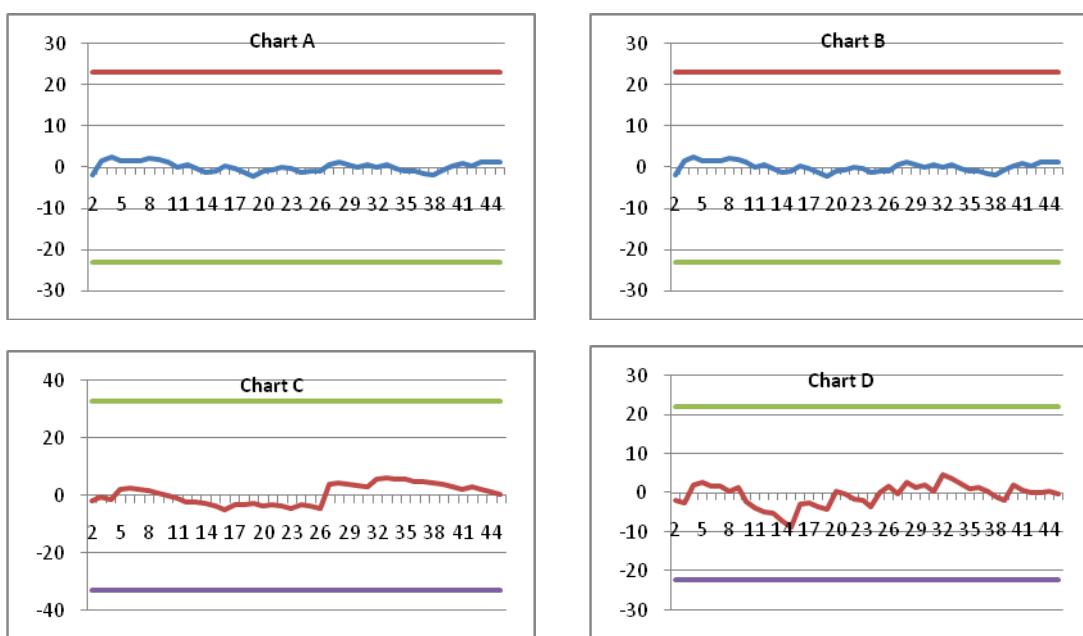
| | | | | | | | | | | | | |
|-----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| BB | 5.23 | 5.14 | 5.05 | 4.95 | 4.87 | 4.78 | 4.69 | 4.61 | 4.52 | 4.44 | 4.36 | 4.28 |
| CC | 9.40 | 9.38 | 9.36 | 9.35 | 9.33 | 9.31 | 9.29 | 9.28 | 9.26 | 9.24 | 9.22 | 9.21 |
| DD | 8.79 | 8.77 | 8.75 | 8.72 | 8.70 | 8.68 | 8.66 | 8.63 | 8.61 | 8.59 | 8.57 | 8.54 |
| EE | 6.69 | 6.67 | 6.64 | 6.62 | 6.59 | 6.56 | 6.54 | 6.51 | 6.48 | 6.46 | 6.43 | 6.41 |
| FF | 3.97 | 3.90 | 3.82 | 3.75 | 3.67 | 3.60 | 3.53 | 3.47 | 3.40 | 3.33 | 3.27 | 3.21 |
| GG | 9.68 | 9.91 | 10.14 | 10.37 | 10.61 | 10.85 | 11.10 | 11.36 | 11.62 | 11.89 | 12.17 | 12.45 |
| HH | 3.37 | 3.31 | 3.26 | 3.20 | 3.15 | 3.09 | 3.04 | 2.99 | 2.94 | 2.89 | 2.84 | 2.79 |
| II | 7.32 | 7.30 | 7.27 | 7.25 | 7.23 | 7.21 | 7.19 | 7.17 | 7.14 | 7.12 | 7.10 | 7.08 |
| JJ | 9.24 | 9.18 | 9.11 | 9.05 | 8.99 | 8.93 | 8.87 | 8.81 | 8.76 | 8.70 | 8.64 | 8.58 |
| KK | 27.72 | 29.39 | 31.15 | 33.01 | 34.99 | 37.09 | 39.31 | 41.67 | 44.17 | 46.81 | 49.62 | 52.59 |
| LL | 11.67 | 11.92 | 12.18 | 12.45 | 12.72 | 13.00 | 13.28 | 13.58 | 13.87 | 14.17 | 14.48 | 14.80 |
| MM | 6.76 | 6.84 | 6.92 | 7.00 | 7.08 | 7.16 | 7.25 | 7.33 | 7.42 | 7.51 | 7.60 | 7.69 |
| NN | 10.15 | 10.21 | 10.27 | 10.32 | 10.38 | 10.44 | 10.49 | 10.55 | 10.61 | 10.67 | 10.73 | 10.79 |
| OO | 8.76 | 8.71 | 8.67 | 8.62 | 8.58 | 8.53 | 8.49 | 8.44 | 8.40 | 8.35 | 8.31 | 8.26 |
| PP | 11.55 | 11.48 | 11.41 | 11.34 | 11.28 | 11.21 | 11.14 | 11.08 | 11.01 | 10.95 | 10.88 | 10.82 |
| QQ | 5.75 | 5.72 | 5.69 | 5.65 | 5.62 | 5.59 | 5.55 | 5.52 | 5.49 | 5.46 | 5.42 | 5.39 |
| RR | 11.33 | 11.28 | 11.23 | 11.19 | 11.14 | 11.09 | 11.05 | 11.00 | 10.95 | 10.91 | 10.86 | 10.82 |
| SS | 13.43 | 13.59 | 13.75 | 13.92 | 14.08 | 14.25 | 14.42 | 14.59 | 14.76 | 14.94 | 15.11 | 15.29 |

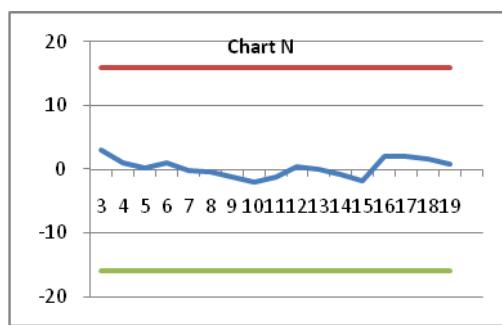
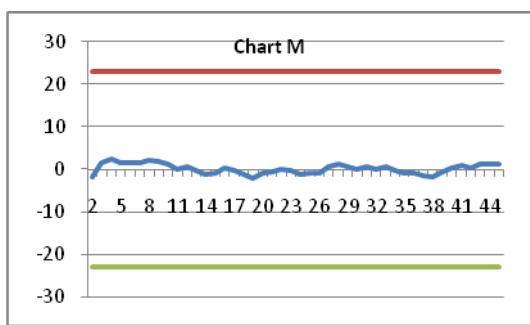
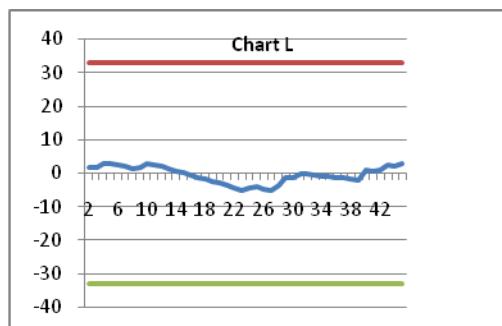
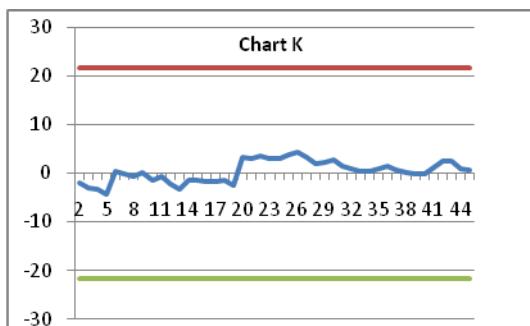
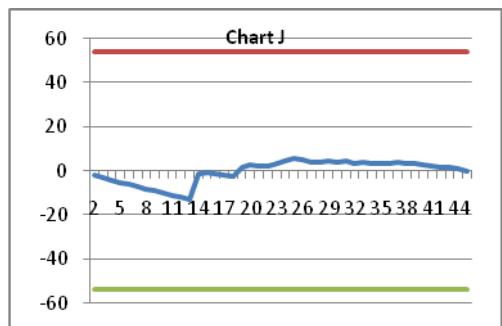
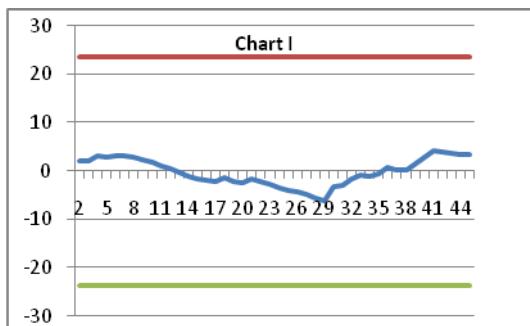
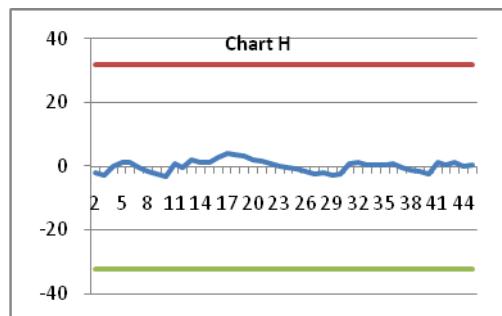
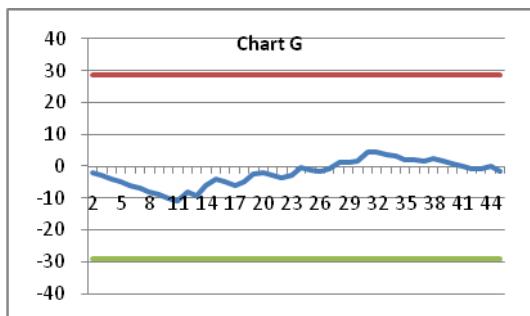
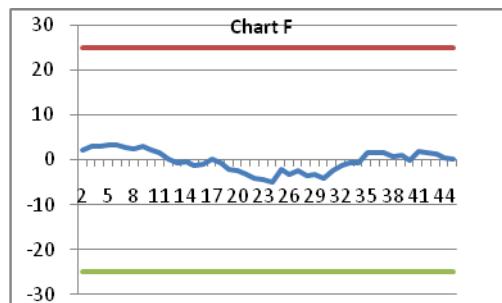
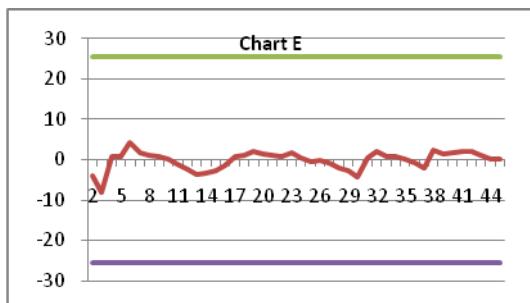
Table 10

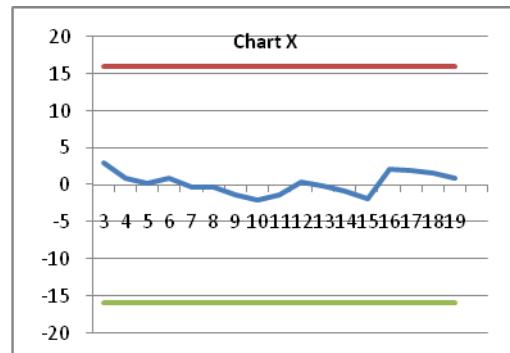
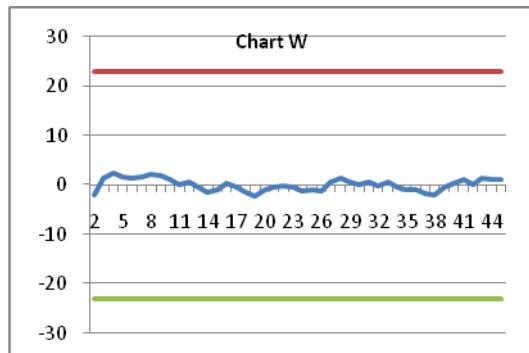
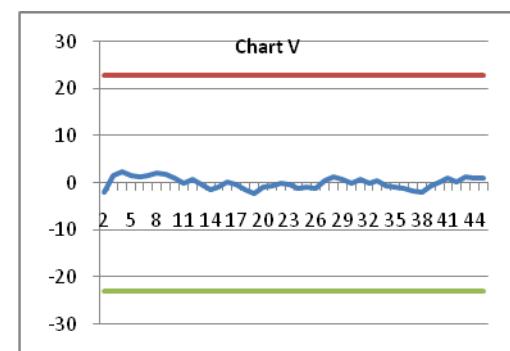
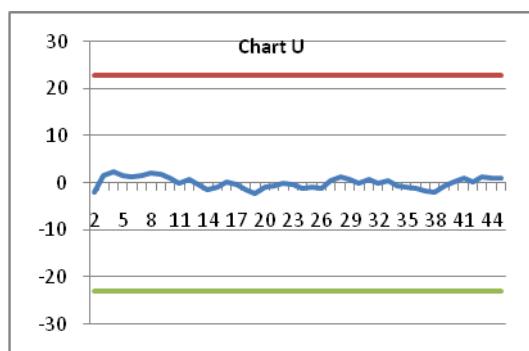
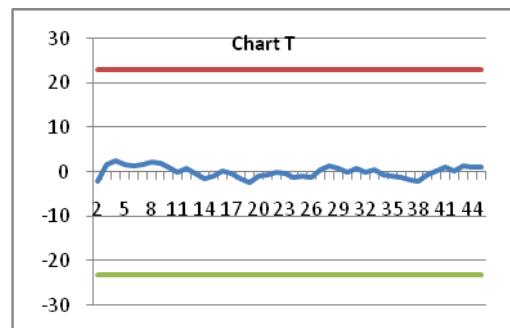
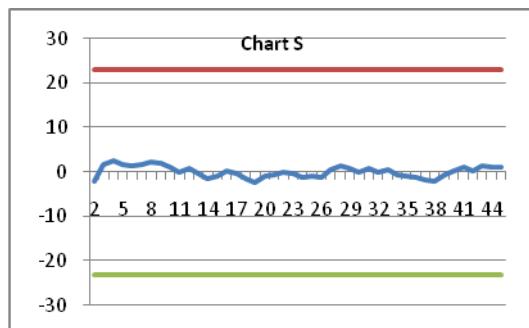
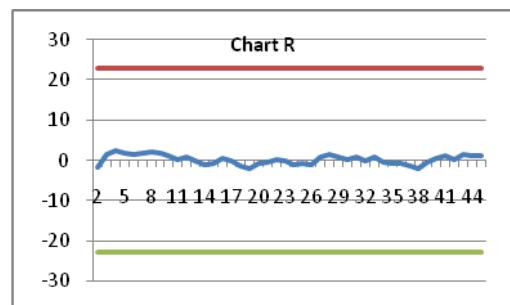
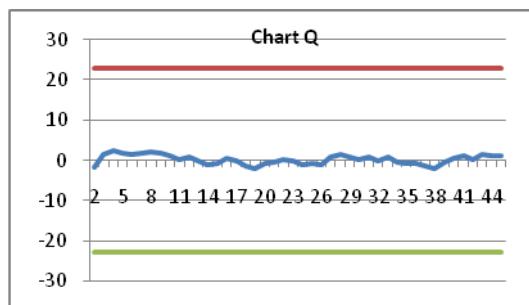
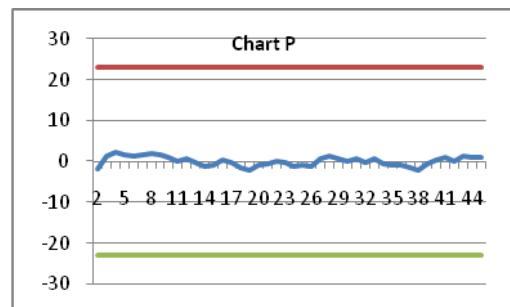
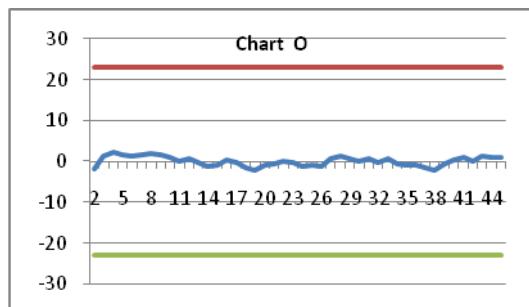
4. CONCLUSIONS

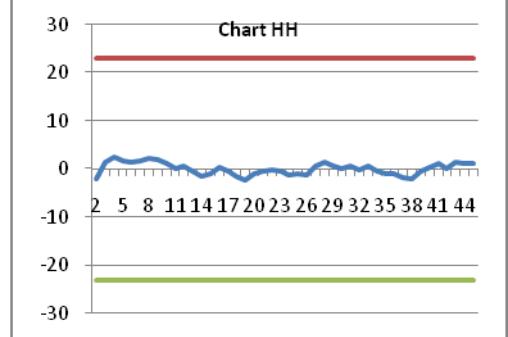
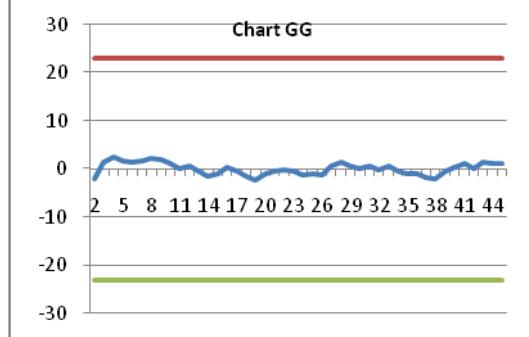
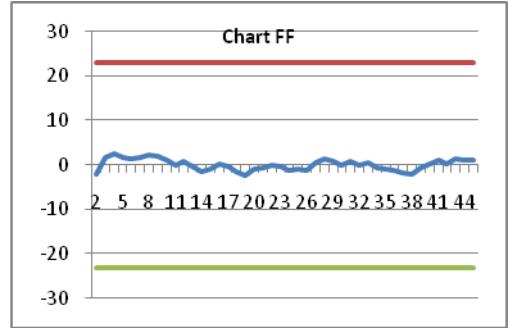
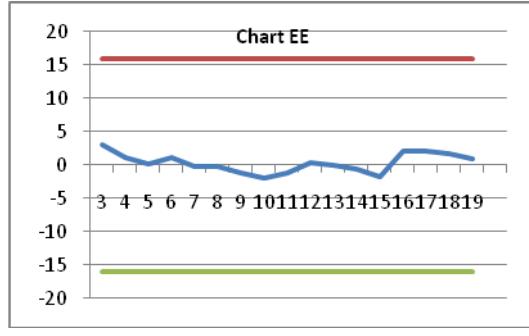
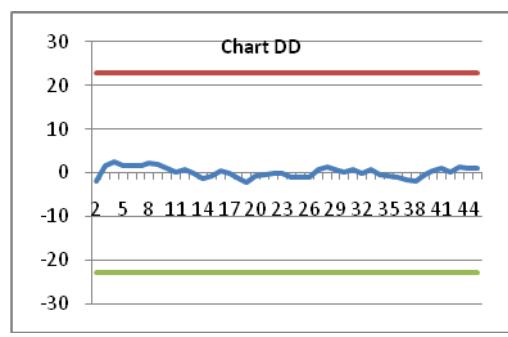
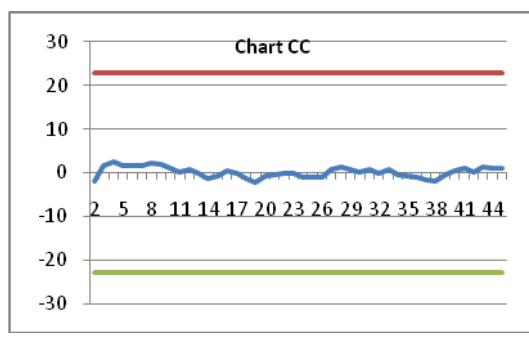
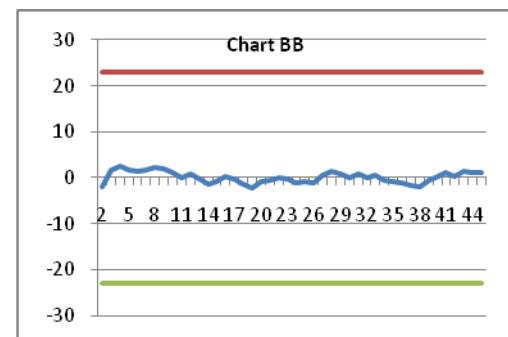
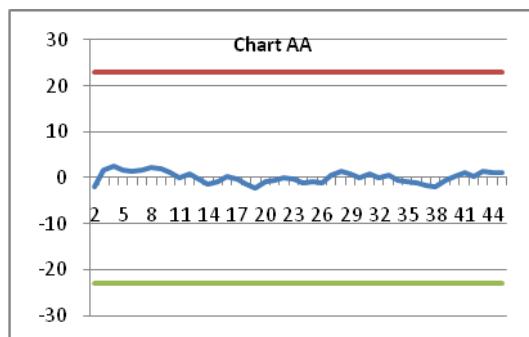
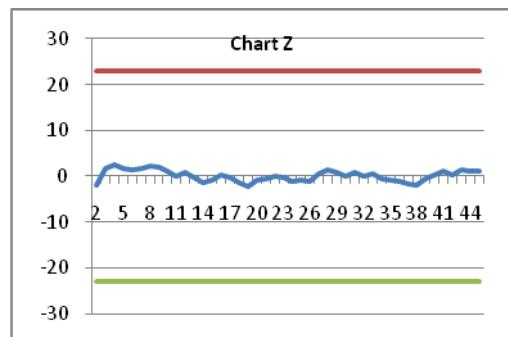
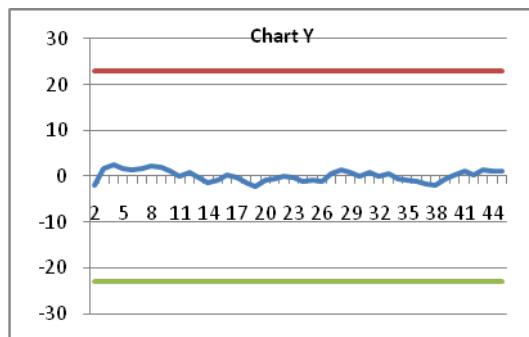
We proposed grey model as an accurate forecasting method for predicting stock market. We choose stock market of Tehran as a data base and gathered information of portfolio's rate of return of 50 companies in stock market which was announced as the best companies last year. At first we computed squares sum of errors with different value of α , which could differ from .1 to .9, to insure that grey model with $\alpha=.5$ is the most appropriate model in forecasting. The results showed just 5 companies from 45 have the less value of errors with $\alpha=.5$, in comparison of model with the other value of α . Then we computed mean squares sum of errors with different type of time series based forecasting methods, which is here, Naïve method, Simple Average method , Moving Average method, single Exponential Smoothing method. The comparison of grey model's errors and the errors of timed series forecasting method confirmed the predictive ability of grey model. by tacking signals we confirmed that grey model forecasting was in control. At the last, portfolio's rate of return computed for next 12 periods.

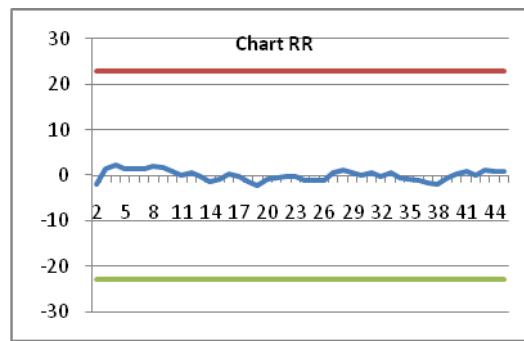
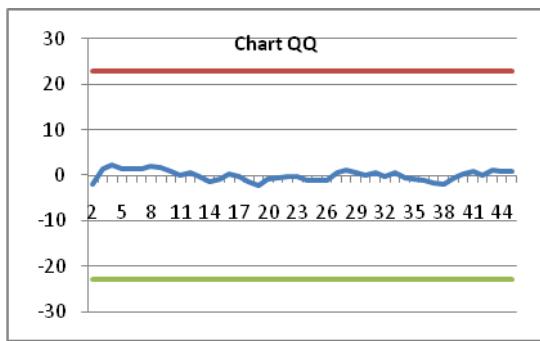
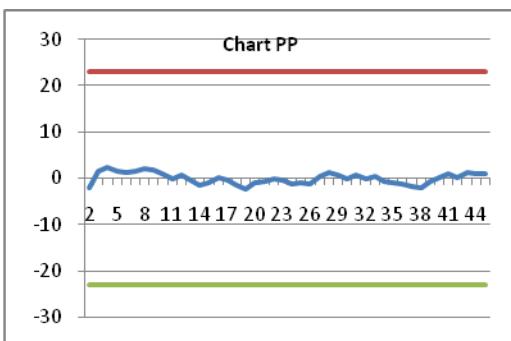
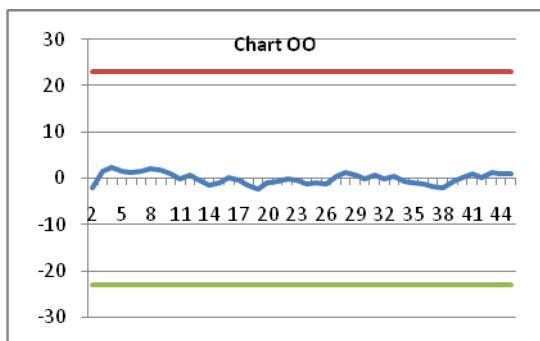
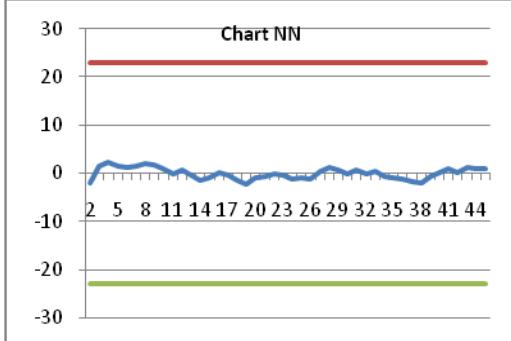
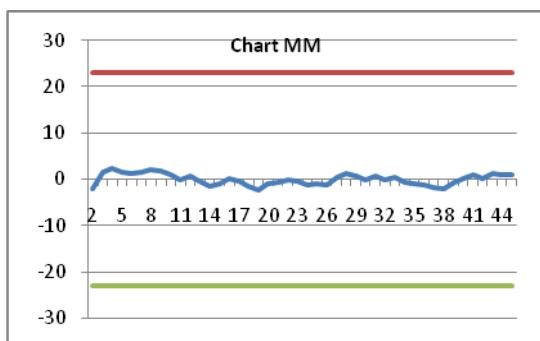
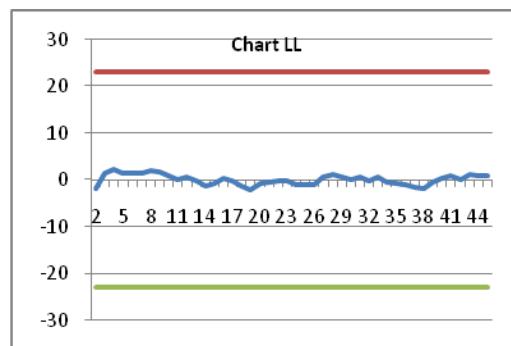
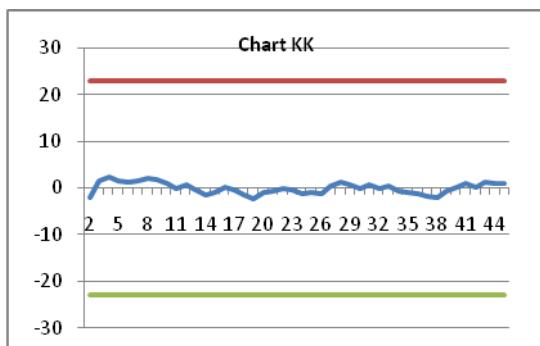
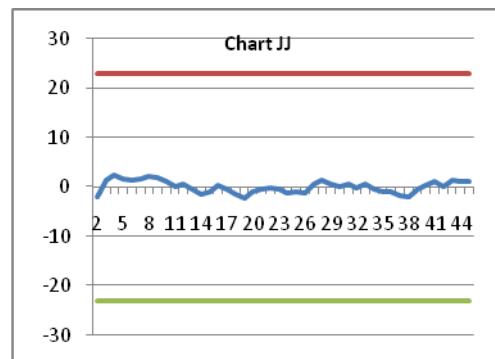
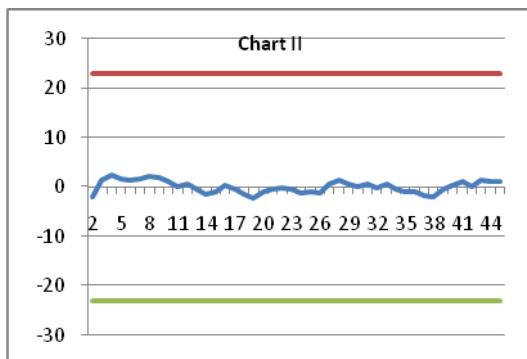
Plots:

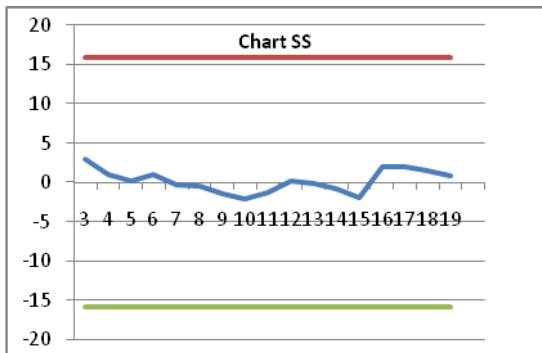












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