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ON AUTOREGRESSIVE DISTRIBUTED LAG, COINTEGRATION AND ERROR CORRECTION MODEL

[An Application to Some Nigeria Macroeconomic Variables]

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ABSTRACT

This paper reviews the use of the traditional ARDL and the ARDL approach to cointegration for the analysis of short-run dynamic and long run relationship when series are difference stationary (series can be integrated of different orders). The two models were used to estimate the short-run dynamics and the long run relationships between selected Nigeria's macroeconomic series. The results compares favorably with the theory that the ARDL is equivalent to the short-run dynamics of the error correction model (the resultant model from the ARDL approach to cointegration).

Keywords: *Autoregressive distributed lag, Cointegration, Error correction, Inflation, Gross domestic product (GDP), Time Series Econometrics.*

1. INTRODUCTION

The future uncertainty of monetary policies requires that policy makers have a good knowledge of where macroeconomic variables, such as inflation and gross domestic product (GDP) are leading in future in relation to other variables so that policies can be engineered to attain desired objectives.

Analyzing indicators to understand how each indicator impact the orders for policy decision making requires the analysis of long-run relationships. In the where the indicators or variables considered in a long-run relationship are trend stationary (i.e. non stationary), the fundamental principle in time series analysis is to de-trend such series and make them stationary. Estimations and inferences concerning the relationships and properties of a series' are then modeled using the Autoregressive distributed lag (ARDL) model. Time Series Analysis sometimes raises the question as to whether it would be useful to consider a related series (say x) when considering another series (say y). If it is felt that series x leads to series y, then one may attempt to build a distributed lag model (Dynamic Regression model) relating the two series. By Distributed lag Model, we mean the inclusion of more than one lag of the regressor in a regression function; the model help to determine the effects of a change in a policy variable over another. The analysis becomes complicated when de-trending is difficult or impossible; in such cases order methods of stationarity are used.

As a basic premise series integrated of order a particular order (i.e. I(d)) or difference stationary cannot be analyzed by the traditional ARDL. Hence, the development of alternative estimation procedure, cointegration. (see Engle and Granger (1987), Johansen (1991), Phillips and Hansen (1990). Cointegration is concerned with the analysis of lon run relations between variables integrated of the same order (i.e. series made stationary at the same order of differencing) and the speed of return to equilibrium after a deviation is measured by the Error Correction Model (ECM). This raises another short fall in analyzing and establishing long run relationships, the cointegration test is not applicable in cases of variables that are integrated of different orders (say, series-A is I(1) and series-B is I(0)). Recent literatures on re-parameterising the ARDL model to the ECM have become the solution to determining the long run relationship between series with different order of integration. The re-parameterized result gives the short-run dynamics (equivalent to the ARDL) and long run relationship of the relationship

This paper explores the issues surrounding the analysis of Cointegration and the Error Correction model within the Distributed Lag model framework, i.e. the Autoregressive Distributed Lag Approach to Cointegration. The re-parameterization is possible because the ARDL is a single model equation and of the same form with the ECM

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In this paper we review the use of the traditional ARDL and the re-parameterized ARDL model to the ECM for the analysis of short-run dynamic and long run relationship when series are difference stationary (series can be integrated of different orders).

The rest of the Paper is organized as follows. The next section reviews the ADRL model specification. Section 3 reviews the re-parameterization of the ARDL to the ECM and we establish that the short-run dynamics of from the re-parameterized ECM is equivalent to the ARDL. Section 4 turns to establishing the relationship between some selected Nigerian macro-economic series using the different procures. A detailed summary is contained in the final section.

2. METHODOLOGY

ARDL Model Specification

An autoregressive distributed lag model is considered as

(ARDL(1,1) model: $y_t = \mu + \alpha_1 y_{t-1} + \beta_o x_t + \beta_1 x_{t-1} + u_t$

Where y_t and x_t are stationary variables, and u_t is a white noise.

Generalizations:
Using the lag operator L applied to each component of a vector, $L^k x_t = x_{t-k}$, it is easy to define the lag polynomial A(L) and the vector polynomial B(L)
The ADRL(p,q) model:

$$A(L)y_t = \mu + B(L)x_t + u_t,$$

With

$$A(L) = 1 - \alpha_1 L - \alpha_2 L^2 - \dots - \alpha_p L^p,$$
$$B(L) = 1 - \beta_1 L - \beta_2 L^2 - \dots - \beta_q L^q.$$

Hence, the general ARDL(p, q_1, q_2, …, q_k) model:

$$A(L)y_t = \mu + B_1(L)x_{1t} + B_2(L)x_{2t} + \dots + B_\kappa(L)x_{kt} + u_t.$$

If A(L) = 1, the model becomes a distributed lag model (no lags of y_t).

Estimation:
If the values of x_t are treated as given, as being uncorrelated with u_t. OLS would be consistent. However, if x_t is simultaneously determined with y_t and $E(x_t,u_t) \neq 0$, OLS would be inconsistent. As long as it can be assumed that the error term u_t is a white noise process, or more generally, is stationary and independent of x_t, x_{t-1}, … and y_t, y_{t-1}, …, the ARDL models can be estimated consistently by ordinary least squares.

3.0 RE-PARAMETERIZATION TO THE ECM:

We write the ADRL model as a lag polynomial in y as,

$$y_t = (1 + \alpha_1 + \alpha_1^2 + \square)\mu + (1 + \alpha_1 L + \alpha_1^2 L^2 + \dots)(\beta_o x_t + \beta_1 x_{t-1} + u_t)$$

The current value of y depends on the current and all previous values of x and u.

$$\delta y_t/\delta x_t = \beta_o, \quad \text{[impact multiplier]}$$

The effect of the first lag (after one period),

$$\delta y_{t+1}/\delta x_t = \beta_1 + \alpha_1\beta_o,$$

Effect at second lag,

$$\delta y_{t+2}/\delta x_t = \alpha_1\beta_1 + \alpha_1^2\beta_o,$$

Since y_t is equivalent to $y_{t-1} + \Delta y_t$ and x_t is equivalent to $x_{t-1} + \Delta x_t$, we substitute y_t and x_t with $y_{t-1} + \Delta y_t$ and $x_{t-1} + \Delta x_t$.

Therefore, we have,

$$\Delta y_t = m + \beta_o\Delta x_t - (1 - \alpha_1)y_{t-1} + (\beta_o + \beta_1)x_{t-1} + u_t,$$
$$\Delta y_t = \beta_o\Delta x_t - (1 - \alpha_1)[y_{t-1} - (m/(1 - \alpha_1)) - ((\beta_o + \beta_1)/(1 - \alpha_1))x_{t-1}] + u_t,$$

If we let: $\varphi_0 = (1 - \alpha_1)$,
$\varphi_1 = (\beta_o + \beta_1)$

We can rewrite the equation in the error correction form as

$$\Delta y_t = m + \beta_o\Delta x_t - \varphi_0[y_{t-1} - \varphi_1 x_{t-1}] + u_t,$$

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And the total long tern effect/long run multiplier (equilibrium), say k_1 is therefore:

$$k_1 = [\varphi_1/-\varphi_0]$$

Y and X will be in their long term equilibrium state when $y = k_0 + k_1x$, where $k_0 = [m/-\varphi_0]$

This is called the error correction model (ECM).

In summary the ECM concludes that the current change in y is the sum of two components.

- The current change in y is proportional to the current change in x
- The current change in y is a partial correction for the extent to which the lag of y (i.e. y_{t-1}) deviates from the equilibrium values corresponding to x_{t-1} (the equilibrium error).

Hence, by differencing and forming a linear combination of the non-stationary data, all variables in an ARDL model are transformed equivalently into an ECM with stationary series only.

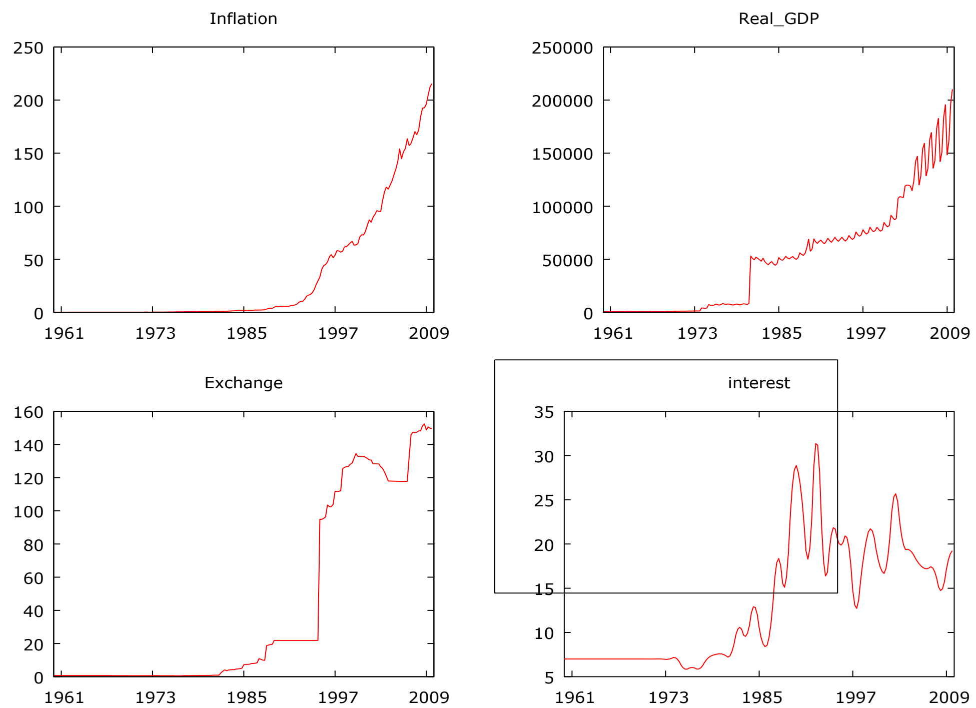
3.1 Data

The data used in this study were the Nigeria's quarterly Real GDP, Inflation rate and Exchange Rate Collected from the Central Bank of Nigeria's Statistical Bulletin 2010 while the data for the Interest rate used is reported on annual basis, this then required the disaggregation of the Interest rate data into a quarterly series using the FLOW Denton method from the ECOTRIM Software.

4.0 RESULTS AND DISCUSSION

The Descriptive Statistics for the three time series show that the Nigerian Inflation rate was stable between 1960 to just before 1982 and since then has been on the increase with about 10% every quarter. The Real GDP shows that there has been a rapid growth in the series with some level of shocks, which coincides with the Country's transition in power from Military Regime to the Civilian administration. The Real GDP also reveals a level of seasonality with the data exhibiting a large increase in about every ten years. Also the Exchange rate shows a steady moving at the initial stage but grew rapidly between 1977 and 1994, which happens to be the time General Sani Abacha took control of power and the Exchange Rate grew to about 450% increase in the first quarter of 1995, it increases steadily and decrease to a point where it remained stable (before the current increase being experienced) for about 3 years (between 2004:1 to 2006:3), this stability could be associated with the international organizations Debt relieve for the Nation. The Interest rate shows an unsteady growth in the series, the growth could be associated to various political and economical reasons. The Doornik-Hansen test and Jarque Bera Test (Jarque and Bera, 1981) statistics suggest that the null hypothesis of normality should be rejected for all four time series. (Results tables are in Appendix 1)

Figure 1: Time Plots of Inflation, Real GDP, Exchange and Interest Rates



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4.2 Unit Root Tests

From Appendix 1, the ADF unit root test shows that all the series are non-stationary at 5% level of significance except the interest rate. However, the non-stationary series' attained stationarity after the first difference.

This imply that the Nigerian Real GDP, Inflation and Exchange rates are integrated of order one, I(1), while interest rate is integrated of order zero, I(0).

4.3 Autoregressive Distributed Lag

Using the ARDL model:

$$\Delta y_t = \sum_{i=1}^{p} a_i \Delta y_{t-i} + \sum_{i=0}^{n} c_i' \Delta x_{t-i} + e_t$$

where Δ represents the first difference of the variables, p and n are the lag lengths and the e_t is a scalar mean error term.

4.3.1 INFLATION

Table 1: Autoregressive Distributed Lag

	coefficients	Coefficient	std. error	t-ratio	p-value
d_Inflation_1	0.1473	0.0654	2.249	0.0257	**
d_Inflation_4	0.5165	0.0644	8.02	1.30E-13	***
d_Real_GDP_3	7.43E-05	2.60E-05	2.86	0.0047	***
d_Exchange_1	0.0519	0.0254	2.042	0.0426	**
interest_2	0.3323	0.1906	1.743	0.0831	*
AIC = 24.22	BIC = 24.72	HQC = 24.42	Portmanteau test = 188.98 (df = 176, p-value 0.238496)		

The result above shows that there are significant effects of the lags of some of the macroeconomic variables on Inflation. We have a significant effect of the third lag of Real GDP, first lag of Exchange Rate and the fourth and second lag of Interest rate also the first and fourth lagsof inflation have significant effect on the inflation rate, implying that the current Real GDP rate would still affect the rate of Inflation in the next 3 year, the current Exchange rate would affect the Inflation rate for the coming year and the current Inflation Rate would still have an influence on the inflation rate in the next four year.

4.3.2 Real_GDP

	coefficient	coefficient	std. error	t-ratio	p-value
d_Inflation_4	392.7600	169.5640	2.3160	0.0217	**
d_Inflation_5	559.4730	197.9010	2.8270	0.0053	***
d_Inflation_6	-355.7400	205.2870	-1.7330	0.0849	*
d_Real_GDP_1	-0.1813	0.0756	-2.3990	0.0175	**
_Real_GDP_2	-0.3317	0.0758	-4.3760	0.0000	***
d_Real_GDP_3	-0.2254	0.0744	-3.0310	0.0028	***
d_Real_GDP_4	0.4465	0.0791	5.6420	0.0000	***

*** significant at 0.01 ** significant at 0.05 * significant at 0.1

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The result above shows that there is a significant effect of the third lag of Real GDP, first lag of Exchange Rate and the fourth and fifth lag's of Inflation on the inflation rate, implying that the current Real GDP rate would still affect the rate of Inflation in the next 3 year, the current Exchange rate would affect the Inflation rate for the coming year and the current Inflation Rate would still have a influence on the inflation rate in the next four and five years.

4.4 Error Correction Model

The unit root s results suggests that the variables are integrated of different orders i.e. Inflation, Real_GDP and Exchange rate are integrated of order one, I(1), while Interest rate is integrated of order 0, I(0). Hence, the ADL approach to cointegration (the re-parameterized model) is appropriate to model the long run relationship between the series'.

4.4.1 Inflation

Inflation	Annual Data		Lev. of sig.	Quarterly Data		Lev. of sig.
	Coefficient	Sig.		Coefficient	Sig.	
Const.	21.49	0.00094	***	-0.41	0.19688	
d_inflation_1	0.42	0.03078	**	-0.0093	0.89803	
d_inflation_2	-0.09	0.56358		-0.07	0.34904	
d_inflation_3	-	-	-	-0.18	0.01068	**
d_inflation_4	-	-	-	0.36	<0.00001	***
d_Real_GDP_1	4.89E-05	0.00204	***	-1.43E-05	0.55909	
d_Real_GDP_2	6.67E-06	0.50663		-1.41E-05	0.56676	
d_Real_GDP_3	-	-	-	5.88E-05	0.01980	**
d_Real_GDP_4	-	-	-	-7.33E-06	0.78712	
d_exchange	-0.21	0.44265		0.015	0.52410	
d_exchange_1	-0.81	0.00971	***	0.06	0.01349	**
d_exchange_2	0.36	0.20828		0.0009	0.97033	
d_exchange_3	0.75	0.01378	**	-0.03	0.21050	
Interest	0.86	0.36041		0.26	0.15182	
interest_1	1.19	0.03047		-0.55	0.10337	
interest_2	2.91	0.01432	**	0.35	0.05769	*
d_investment	0.001	0.00041	***	-	-	-
d_investment_1	0.0005	0.14646		-	-	-
d_investment_2	0.0012	0.00256	***	-	-	-
EC1	-1.27	0.00004	***	0.022	0.00004	***

*** significant at 0.01 ** significant at 0.05 * significant at 0.1

The tables indicates the long run equilibrium and short run dynamics of some the Nigeria Economic Variables The Long run coefficients show that in the long run, the coefficient of investment has a significant impact on Inflation [sig. = 0.0004], and a one percentage increase in investment leads to a 0.001% increase in Inflation. A one percentage increase in exchange rate leads to 0.21 decreases in inflation. This indicates that exchange does not have an important effect on inflation (inf). In addition, the coefficient of interest rate is not statistically significant at the 5 percentage level. As discussed, the error correction term indicates the speed adjustment to restore equilibrium in the dynamic model. The ECM coefficient shows how quickly variables converge to equilibrium and it should have a statistically significant coefficient with a negative sign. According to Bannerjee et al. (1998), the highly significant error correction term further confirms the existence of a stable long-run relationship. The annual result (table below) for Inflation shows that the expected negative sign of EC1 is highly significant. This confirms the existence of the long run relationship among the variables with their various significant lags. The coefficient of EC1 = -1.27, imply that deviation from the long-term growth rate in inflation rate is corrected by 127% by the following year.

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4.4.2 Real GDP

GDP	Annual Data Coefficient	Sig.	Lev. of sig.	Quarterly Data Coefficient	Sig.	Lev. of sig.
Const.	102804	0.25150		587.971	0.49718	
d_inflation_1	3840.88	0.20155		323.189	0.10865	
d_inflation_2	6905.83	0.02555	**	-366.677	0.06065	*
d_inflation_3	-	-		-148.649	0.44206	
d_inflation_4	-	-		319.716	0.10464	
d_Real_GDP_1	0.258996	0.24890		-0.188	0.00582	***
d_Real_GDP_2	0.342901	0.05049	*	0.398	<0.00001	***
d_Real_GDP_3	-	-		-0.258	0.00025	***
d_Real_GDP_4	-	-		0.478	<0.00001	***
d_exchange	9746.82	0.04062	**	21.509	0.74863	
d_exchange_1	11408.1	0.02265	**	-41.503	0.53717	
d_exchange_2	-20078.7	0.00050	***	-49.287	0.47129	
d_exchange_3	26625.6	0.00003	***	-4.429	0.94793	
Interest	34318.5	0.03738	**	-13.136	0.94910	
interest_1	1913.6	0.91864		232.316	0.80322	
interest_2	9227.66	0.60156		-212.157	0.67267	
d_investment	17.306	0.00043	***	-	-	-
d_investment_1	3.96496	0.48541		-	-	-
d_investment_2	2.9534	0.61020		-	-	-
EC1	-100.48.2	0.01519	**	19.274	0.18951	

*** significant at 0.01 ** significant at 0.05 * significant at 0.1

The result for annual GDP (table below) reveals a high percentage of corrected disequilibrium (100.5%) in the previous year corrected for the current year while the quarterly result shows no long run equilibrium of GDP with the other macro-economic variables. The result indicating the non-significance of the exchange rate on GDP is worth mentioning. The non significance of the Exchange rate in the quarterly data can be linked to the fact that the Nigerian system imposes fixed value of exchange rate at the beginning of every year and if this value change in the during the year, the change does not immediately take effect on the GDP. From the annual result it can be concluded that any change in the course of any year will only start having an influence on GDP after the year the change is made. Hence the result from the quarterly data should not be used in any case for policy decision making, except for variables that are known to have an almost immediate influence on GDP (i.e. variables like Inflation and lags of GDP).

5.0 CONCLUSION

The relationship between the selected Nigerian macroeconomic variables showed that while the influence of investment is significant and important in making inflation rate better with the use of its current rate to explain inflation, past rates of inflation, interest, exchange rates and GDP are also important factors to be considered. For GDP, exchange rate is highly influential in determining GDP, investment, interest, GDP and ination rates are also useful. Therefore, making policy decisions should involve both present and past rates of other associated macroeconomic variables. The short term dynamics of the error correction model was observed to be equivalent with the result from the ARDL. Again the error correction model for the annual data indicates long run relationship exist between Inflation, GDP and other macroeconomic series.

The inflation result reveal that the deviation from the long-term state will be corrected by 127% the following year. Also deviation from the long-term growth rate in GDP reveals a high percentage of corrected disequilibrium of 100.5% in the previous year corrected for the current year (i.e. disequilibrum corrected for the next year)

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Finally, we applied the ARDL model approach to cointegration to 5 selected Nigeria macroeconomic series considered and found that the Ination and GDP series have long run relationships with the other macroeconomic series in the annual set of data and a disturbing (instability for GDP and no long run equilibrum for ination) result from the quarterly reported data set. The ARDL models were found to be equivalent to the short term dynamics of the ECM, also the performance of the annual data gave a better and interpretable result and these results follow that of Hassler and Wolters (2006).

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